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SPECIFIC HEAT AND THERMAL BOUNDARY RESISTANCE OF LIQUID He^3†

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We have measured the heat capacity of a mixture of powdered cerium magnesium nitrate (CMN) and liquid He^3 at pressures near 14 cm Hg and at temperatures extending from 0.008°K to 0.040°K. When the heat capacity of the CMN is subtracted, the resultant heat capacity is found to be linear in T within experimental scatter. No evidence has been found for a transition to the highly correlated phase predicted theoretically.[1] From the heat capacity data we derive a value of 2.35 ± 0.20 for the ratio of the effective mass of the quasi-particles to the mass of the He^3 atom. We have also measured the rate of transfer of heat across the interface between the liquid He^3 and its container and find that this rate is proportional to the differences in the fourth powers of the temperatures as expected from the simple picture of the exchange of phonons between two reservoirs at different temperatures.

The He^3 calorimeter had the following characteristics: (i) The cell containing the He^3 was cylindrical in shape, of 1.002-cm length and 0.905-cm diameter at room temperature. (ii) 300 0.004-in. diameter, 99.999% pure, enameled copper cooling wires were embedded in the inside surface of the cell, which was made using Epibond 100A.[2] (iii) He^3 was introduced by means of a 1/64-in. o.d. × 0.003-in. wall Cu-Ni (70-30) tube. (iv) The heater was a 50-ohm, 1½-in. long bifilar lead of 0.002-in. diameter "Evanohm"[3] wire coiled inside the cell. Current was conducted to the heater using 0.003-in. diameter niobium leads. (v) The cell was filled with 0.679 g of loosely packed CMN powder with sizes lying primarily in the range from 0.003 in. to 0.015 in. (vi) The cell was closed by a plug of Epibond 100A and sealed by means of a very small quantity of Epibond 104.[2] The amount of He^3 in the cell below 1°K was measured to be 0.00777 mole with an estimated accuracy of 3%. From the data of Sherman and Edeskuty,[4] the free volume of the cell was 0.294 cm^3 in reasonable agreement with the volume which was calculated on the basis of the geometry when thermal contraction and the volume of the CMN were considered.

The susceptibility of the CMN was measured using a ballistic galvanometer. It was possible to calibrate the very small amount of CMN by comparing the deflection due to the CMN with that due to the chrome alum refrigerator at a variety of temperatures from 4°K to 0.3°K under thermal equilibrium conditions and with the measuring coil and vacuum case at a constant temperature. It was essential to keep the glass vacuum case at constant temperature during calibration since its effective susceptibility was equal to that of the CMN. We assumed that Curie's law holds for CMN in the present temperature range and that, because of the nearly spherical shape of the cell, the magnetic and absolute temperatures were equal to one another. The resultant temperature calibration of the CMN was estimated to have a random error of 2%. Systematic errors due to extraneous magnetic materials or to a shift in the zero of our mutual inductances would decrease the calculated temperatures.

The He^3 was cooled: first, by a He^3 refrigerator; second, by 35.8 g of chromium potassium alum thermally coupled as tightly as possible to the cell; and third, by the CMN itself. No lead switches were used. The characteristics of this part of the apparatus will be published elsewhere.[5]

The He^3 gas was the same as that used in the diffusion and susceptibility measurements of Anderson, Hart, and Wheatley.[6] This gas had a He^4 content of less than 3 parts in 10^5. We have subsequently measured the tritium contamination of the gas. The tritium decay rate in the present cell was measured to be 300 disintegrations/sec, giving a heating rate of 2×10^{-4} erg/min.[7]

After the He^3 was initially cooled, the ballistic deflection was measured as a function of time. From time to time heat was added for short intervals. The resulting shifts in the extrapolated cooling (or warming) curves were used to obtain the temperature changes produced by adding heat. Most of the uncertainty in the data and also some systematic errors came from uncertainty in this extrapolation. The curves without external heat supplied were used to obtain the heat transfer rate across the cell-liquid He^3 interface. Even below 0.01°K the thermal equilibrium time constant of heater-He^3-CMN system was less than 100 seconds. Above 0.015°K equilibrium was attained in 20 to 30 seconds. It was this rapidly attained internal equilibrium which enabled the measurements to be made even with the tightest possible thermal coupling to the chrome alum. This tight coupling, however, did limit the region where we were able to obtain good experimental data to below 0.04°K on the high-temperature end and above 0.008°K on the low-temperature end. Temperature differences produced by heating were typically 0.4×10^{-3} °K at 0.01°K, 10^{-3} °K at 0.02°K, and 3×10^{-3} °K at 0.03°K.

We assumed that the measured heat capacity was given by $c'=\gamma T+b/T^2$, where the second term is the contribution of the CMN, and plotted c'/T vs $1/T^3$ to obtain b. The resultant value of b, 0.8 erg K°/g, is in reasonable agreement with the values reported in the literature[8] and was subtracted from the raw data to obtain the results shown on Fig. 1. The straight line through the experimental points is $C=1.50\times10^6T$ erg/(K°)², where C is the heat capacity of the He^3. This corresponds to a specific heat of $(19.3\pm1.6)T$ joule/mole (K°)² for the liquid He^3.

Using the measured number of moles in the sample as given above and a molar volume of 36.5 cm³/mole at a pressure of 14 cm Hg as deduced from reference 4, the value of m^*/m may be computed from the formula

$$\frac{m^*}{m}=\frac{\hbar^2C}{Nk^2Tm}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}, \qquad (1)$$

where m^*/m is the ratio of the quasi-particle mass to the mass of the He^3 atom, N is the number of atoms in the sample, V is the volume, k is Boltzmann's constant, and $\hbar$ is Planck's constant divided by 2π. The result is $m^*/m=2.35\pm0.20$, where the error is estimated from the errors in the cell volume, the calibration of the magnetic thermometer, and the raw data. Possible systematic errors in magnetic thermometry would serve to increase m^*/m. Brewer, Daunt, and Sreedhar[9] obtain $m^*/m=2.00\pm0.05$ with convincing data, but it is possible that the specific heat is not yet linear in T at their lowest temperature, 0.085°K. We hope to revise our ap-

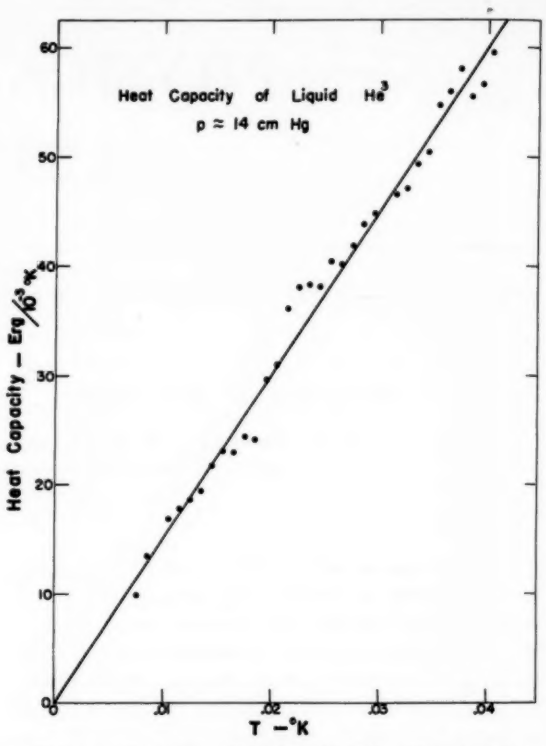

FIG. 1. Heat capacity of liquid He^3. The straight line is $C=1.50\times10^6T$ erg/(K°)². The points represent data averaged over five experimental runs.

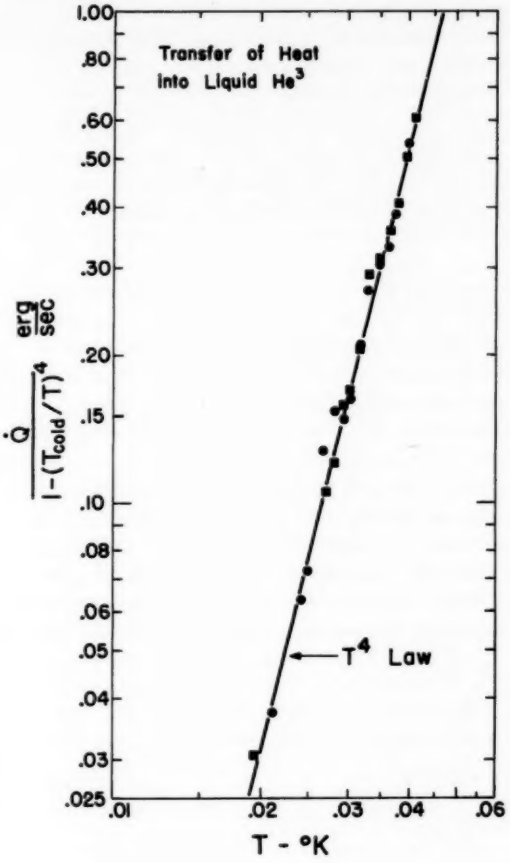

FIG. 2. Logarithmic plot of $\dot{Q}/[1-(T_{cold}/T)^4]$ vs T for the heat flow $\dot{Q}$ between the cell walls and the liquid He^3. The straight line corresponds to a T^4 temperature dependence. ●—First run; ■—second run.

paratus and bridge the gap between the two measurements.

The experimental data for the heat transfer rate between the cell walls and the liquid He^3 are shown in Fig. 2. Heat flow rates $\dot{Q}$ were calculated from the measured heat capacity and the time rate of change of ballistic deflection. The data were treated as follows in order to display the temperature dependence of the heat transfer rate. For T greater than the chrome alum temperature, T_a, we estimated the alum temperature from that temperature region where T was not changing with time. We then plotted $\dot{Q}/[1-(T_a/T)^4]$ vs T logarithmically. All data above 0.022°K were treated in this way. For T less than T_a, we calculated $\dot{Q}/[1-(T/T_a)^4]$ for the same value of T_a as was used for $T>0.022$°K for a given run, and this quantity was constant for $T>0.01$°K. The chrome alum temperatures were adjusted slightly (by 6×10^{-4} °K in one case, 11×10^{-4} °K in the second) to obtain best agreement. The straight line drawn through the points in Fig. 2 represents a T^4 law. As plotted, the ordinate gives the rate of heat transfer from the cell surface at T into He^3 at $T=0$. The equation of the line is $\dot{Q}/[1-(T_{cold}/T)^4]=2.0\times10^5T^4$ erg/sec $(K^\circ)^4=7.2\times10^4AT^4$ erg/sec cm^2$(K^\circ)^4$, where A is the surface area of the cylindrical walls. Although the temperature dependence does not agree with that measured by Lee and Fairbank[10] at higher temperatures, it does agree with the results of the calculation of Bekarevich and Khalatnikov[11] who find the same temperature dependence as we observed. Taking for Epibond 100A a density of 1.17 g/cm^3 and a velocity of sound of 3×10^5 cm/sec as estimated from its elasticity, and substituting into the formulas of reference 11 using known constants for He^3, we find the magnitude of the heat flow predicted theoretically to be about three to six times less than we observed. This is probably reasonable agreement since the velocity of sound in the Epibond 100A is not known with any accuracy.

We wish to acknowledge helpful discussions with Professor John Bardeen. We are grateful to Dr. R. F. Nystrom, who measured the tritium contamination in the He^3; and to G. DePasquali, who very kindly helped us with some chemistry problems.

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*National Science Foundation Fellow.

‡Alfred P. Sloan Fellow.

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HIGH-ENERGY X RAYS DURING SOLAR FLARES*

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The study of energetic particles in space has revealed many interesting phenomena concerning solar cosmic rays, proton streams, plasma clouds, and their interaction with the earth's magnetic field and its atmosphere. Measurements of the ultraviolet and soft x-ray fluxes have led to greater understanding of the processes occurring above the surface of the sun. We present some measurements which are pertinent to further knowledge of solar phenomena, namely, the detection of high-energy x rays coincident with solar flares.

Peterson and Winckler[1] have reported a short-duration x-ray burst (less than 18 sec) at balloon altitudes coincident with a class 2+ flare; neither the energy spectrum nor the time history of the event could be measured with their equipment. Energy spectrum measurements of x rays with energies up to 60 kev during three class 2+ flares have been made with rocket-borne scintillation counters.[2] Since the rockets were launched after the flares had begun, and only remained aloft for short periods, no complete time history of the events was possible. We have detected two x-ray bursts with energies exceeding 80 kev during a class 2 and a class 1 flare; spectrum measurements and the complete time history of these events were obtained. These events were recorded during an October 12, 1960 balloon flight which reached a ceiling altitude of 120 000 feet (4.8 g/cm^2) over southern Arizona.

The instrumentation consists of a NaI scintillation counter which is pointed at the sun by a tracking device described by Shechet.[3] The scintillation counter is encased in a 1-cm thick lead housing which has small holes drilled in the front face so that the field of view of the crystal is approximately a cone of 6° half-angle. The electronics shown in Fig. 1 consist of a transistorized pulse amplifier and a five-channel integral discriminator. The output pulse of each discriminator is shaped to reduce its bandwidth and

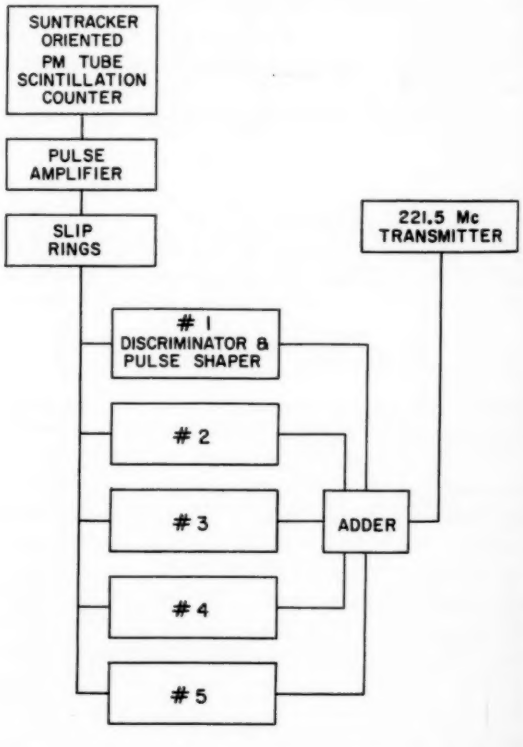

FIG. 1. Diagram of the electronics flown on the balloon.

sent through an adder circuit. The resulting signal, which has five discrete amplitudes, frequency-modulates a 2-watt transmitter. This configuration has the advantages that either differential or integral pulse-height analysis can be made with the ground equipment and all the information can be sent over one telemetry channel. A modified 1680-Mc radiosonde transmitter is used to telemeter altitude, component temperatures, and the pointing error of the sun-tracker.

The data for the two events are shown in Fig. 2. We have presented the integral counting rates for the first three discriminator channels corresponding to energy loss in the crystal of 20, 80, and 150 kev, respectively. Channel 1 was recorded by an Esterline Angus recorder with a 1.5-sec integration time while channels 2 and 3 were recorded by a Rustrak recorder with a 1.5-sec charging time and 7.5-sec discharge time. The background counts are caused mainly by x rays from cosmic-ray induced showers in the atmosphere; approximately one half of the background rate in channel 1 are caused by pulses above 500 kv, the energy threshold of the fifth discriminator. The two events beginning at 1730 UT and 1747 UT shall be referred to as Event 1 and Event 2 hereafter. Event 1 goes off scale on the channel 1 recording. Time markers were put on the charts periodically and are accurate to within 1 sec as timed by *WWV*.

The interesting solar and terrestrial events which occurred during this time were:

(a) A class 2 flare began at 1722 UT, reached a peak at 1730 and at 1738, and ended at 1822.[4]

(b) A class 1 flare began at 1742 UT, reached its peak at 1750, and ended at 1852 UT.[4]

(c) A sudden enhancement of atmospherics (SEA) occurred from 1747 to 1851 UT.[4]

(d) A sudden cosmic noise absorption (SCNA) occurred from 1749 to 1752 UT.[4]

(e) The radio noise storms which occurred are listed in Table I.[5]

Event 1 begins at the peak of the class 2 flare and Event 2 starts in coincidence with an SEA associated with the class 1 flare. Both events are immediately preceded by Type III radio noise bursts. The most plausible process for the production of these high-energy x rays is bremsstrahlung of electrons in the solar atmosphere. It is noteworthy that no x rays were observed during the Type IV radio storm which occurred at 1753-1800 UT, since this type of storm is generally attributed to synchrotron radiation from high-energy electrons. It is possible that these electrons are circling in regions where the

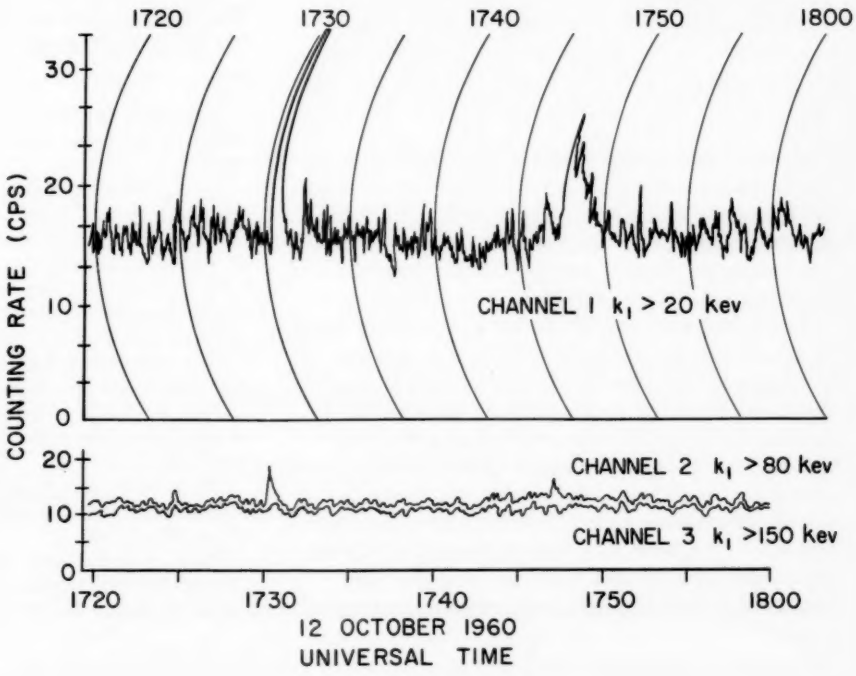

FIG. 2. The counting rate of the first three discriminators as a function of time. The threshold x-ray energy for each channel is given. The time constant for channel 1 is smaller than for the other channels by a factor of 5.

Table I. Solar radio data, 1700 to 1800 UT, October 12, 1960.

Type	Time (UT)	Frequency range (Mc/sec) Harvard	Michigan
III	1725-1730	25-250	
IIIg	1726		100-180
IIIg	1727		100-180
V	1727.5-1728		100-180
IIIg	1729-1730		100-320
IIIg	1745.4-1747	50-500	100-320
Is	1750-2130		100-320
II	1752-1802.2	30-250	100-320
IV	1753-1800	150-400	300-600

density is too low to allow the detection of bremsstrahlung x rays by our equipment.

It is possible to infer something about the electron energy distribution in the solar atmosphere from the spectrum of the bremsstrahlung x rays. If one uses an approximate nonrelativistic bremsstrahlung cross section,[6]

$$\phi d\left(\frac{k}{T}\right) \approx \frac{5}{137} Z^2 \left(\frac{e^2}{mc^2}\right)^2 \frac{mc^2}{T} \frac{dk}{k}, \qquad (1)$$

where k is the energy of the photon, T is the kinetic energy of the electron, and the other symbols are standard, it can be shown that the counting rate at the top of the atmosphere for a detector whose threshold photon energy is k_1 is

$$R(k_1) \propto \int_{k_1}^{\infty} \log(T/k_1) \frac{n(T)dT}{T^{1/2}}. \qquad (2)$$

Here $n(T)$ is the differential energy spectrum of the electrons. We find that Event 2 corrected to the top of the atmosphere gives $n(T) \propto T^{-1.6}$ if a power law spectrum is assumed. No calculation was made for Event 1 because of the off-scale reading on the first recorder.

If one assumes that the decay of the x-ray burst is due to energy loss by collision with neutral hydrogen rather than due to the acceleration mechanism, or other processes, one obtains a neutral hydrogen density $\approx 10^{10}$ atoms/cm^3 for the 100-sec decay time which characterizes Event 2. Densities of this order are found in the chromosphere about 4000 km above the photosphere.[7] The peak intensity of Event 2 at the top of the atmosphere was 1.2×10^{-6} erg/cm^2 sec while that of Event 1 was estimated to be larger by a factor of 2.

It will be interesting in future measurements to see if the pattern of Type III bursts preceding the x-ray bursts persists. Both events observed on this flight show a decay time slower than the rise time; the onset of both events is at least as fast as the recording equipment used in these measurements.

A balloon flight made on August 17, 1960, carrying the same instruments described here was at ceiling altitude of 117 000 feet (5.2 g/cm^2) during a class 1 flare. No increase in counting rate was observed. There were no terrestrial effects or radio noise storms observed during this flare. Future flights are planned and the energy region between 20 and 100 kev will be examined more closely to obtain better spectrum measurements.

The authors would like to thank Miss Dorothy Trotter of the High Altitude Observatory for notifying us of suitable times to make balloon flights, Mr. Hap Owens and his crew at the U. S. Weather Bureau Station in San Diego for tracking and recording our radiosonde data, Colonel Walker and Lieutenant Colonel Bowman at the U. S. Army Test Station, Yuma, Arizona, for providing a balloon launch site, quarters, and the use of their Rawin tracking unit. We also acknowledge the services of many people in our laboratory who take time from their own work to assist in balloon launchings and especially to Dr. W. C. Erickson for many interesting discussions.

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RECOMBINATION IN A HELIUM PLASMA*

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Experiments have shown that electron-ion recombination in gas discharges usually proceeds at a rate well in excess of the direct radiative rate. In weakly ionized discharges the experimental results appear to be moderately well explained by the process of recombination via the dissociation of molecular ions.[1] However, the rapid recombination observed in highly ionized discharges[2] appears difficult to explain by the molecular dissociation process. D'Angelo has recently suggested[3] that the dominant recombination mechanism in highly-ionized, low-temperature plasmas may be the inverse process of collisional ionization, i.e., the capture of an electron by an ion in a collision between the ion and two electrons. In this Letter we present experimental confirmation of the dominance of this three-body recombination in a low-temperature helium plasma.

Measurements were performed on magnetically confined plasmas in the B-1 stellarator,[4] the vacuum tube of which is 450 cm long and 4.8 cm in diameter. The plasmas were produced by ionizing helium gas with a 150-volt, 20-kc/sec electric field induced around the plasma loop. With this mode of excitation it was possible to achieve a high degree of ionization, from almost 100% to 2% as the helium pressure was varied from 0.25 to 100 microns. After removal of the breakdown voltage the following properties of the decaying helium plasma were measured: (1) the electron density, from the phase shifts of 8.6- and 4.3-mm microwaves propagated transverse to the plasma column; (2) spectroscopic measures of the absolute intensity of emission of each line in the visible neutral helium spectrum; (3) relative variations in the intensity of the total light; and (4) the electron temperature, from impedance measurements[5] with a low-power 60-kc/sec signal induced around the plasma loop.

After removal of the breakdown voltage the electron temperature drops rapidly below one volt; then ionization by electron impact becomes negligible and the plasma loss is governed by diffusion and recombination. The electron density was observed to fall exponentially between 5×10^{13} and 3×10^{11} electrons cm^{-3}. The intensities of the individual spectral lines and the total light intensity decayed exponentially at the same rate. The electron temperature fell at a somewhat slower rate. As shown by the experimental data in Fig. 1, the plasma loss rate is roughly proportional to the square root of the neutral

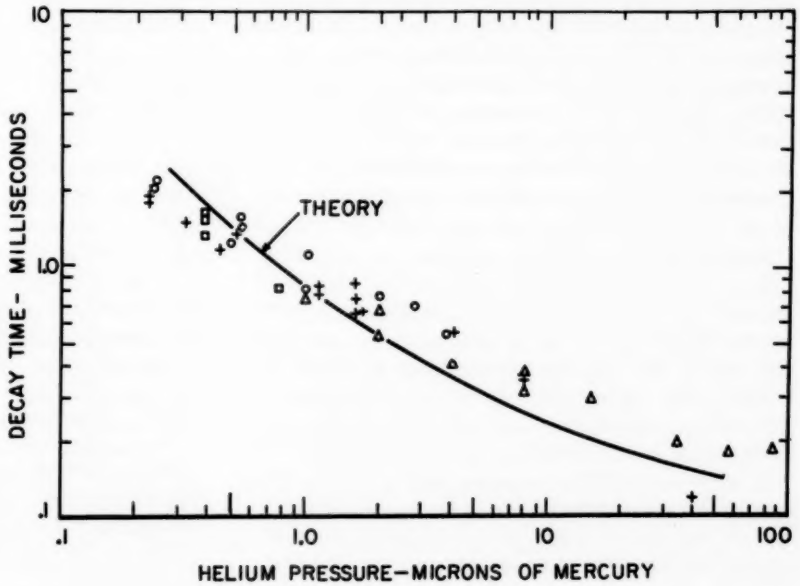

FIG. 1. Time constants for the initial decay of electron density in the stellarator following discharges in helium. The symbols represent experimental data obtained in a series of separate runs with the B-1 and B-3 stellarators.

pressure between 0.25 and 50 microns.

We believe that the plasma loss is caused primarily by recombination rather than by diffusion, because (1) the loss rate is independent of the confining magnetic field between 29 and 3.5 kilogauss; (2) the intensity of the light, which was shown by spectral analysis to originate from the recombining helium atoms, is proportional to the electron loss rate, independent of pressure and magnetic field; (3) the absolute intensity of the spectral lines accounts for substantially all of the electrons which disappear.

These basic features of the recombination process can be derived from the three-body recombination process. We have computed the capture rates for helium, taking into account all $\Delta l = -1$ radiative transitions. Collisional transitions between excited states have been neglected. The results show that the capture rate is a function only of the electron density, n, and the electron temperature, T, and can be approximately represented in the range of variables $10^{11} < n < 5\times10^{13}$ and $0.03 < kT < 0.3$ ev by the expression

$$dn/dt = 0.7\times10^{-19} n^{2.5}/(kT)^3, \qquad (1)$$

with n in $(\mathrm{cm})^{-3}$, t in sec, and kT in ev.

To predict the plasma loss rate one must know the time variation of the electron temperature. Since the electrons and the ions are almost in thermal equilibrium, the plasma temperature is determined by the interplay between two competing processes: energy loss via ion-neutral charge exchange and energy gain from metastable atoms formed by the recombination. The latter arises from the possibility that an atom in the $2\,^3S$ metastable level may undergo a superelastic collision with a plasma electron, returning 19.8 ev to the plasma. The time variation of the plasma energy can be written

$$3d(nkT)/dt = -n_0 n\sigma_i v_i \tfrac{3}{2}k(T - T_0) + n_m n\sigma_e v_e \epsilon, \quad (2)$$

where n_0 and n_m are the neutral and the metastable densities, σ_i and σ_e are the cross sections for charge exchange[6] and metastable destruction,[7] v_e and v_i the electron and ion velocities, and ϵ the de-excitation energy of the helium metastable atom. If the gas pressure is low (< 4 microns), the metastables flow freely to the walls, where they are destroyed. Under these conditions, if three-fourths of the electrons recombine to the $2\,^3S$ metastable level, the metastable density is given by

$$n_m = \tfrac{3}{4}\frac{dn}{dt}\frac{1}{n\sigma_e v_e + v_i/d}, \qquad (3)$$

where d is the average flight path, equal to the tube diameter. If the gas pressure is high (> 15 microns), the metastables diffuse to the walls. Then

$$n_m = \tfrac{3}{4}\frac{dn}{dt}\frac{1}{n\sigma_e v_e + (0.48 v_0/d^2 n_0 \sigma_k)}, \qquad (4)$$

where σ_k is the metastable diffusion cross section.[8] The equations were integrated numerically, with the cross sections taken as $\sigma_i = 3\times10^{-15}$ cm^2, $\sigma_e = 8\times10^{-17}$ cm^2, $\sigma_k = 3.4\times10^{-15}$ cm^2. A comparison of the time dependence of the computed and measured electron density and temperature is given in Fig. 2. The analysis predicts successfully the initial exponential time variation of the electron density, the variation of this exponential with gas pressure (as shown in Fig. 1), and the break in the electron density curve when the electron temperature falls to room temperature. The absolute agreement must be considered fortuitous, since uncertainties in the cross sections, in the reflection of metastable atoms from the stainless steel walls,

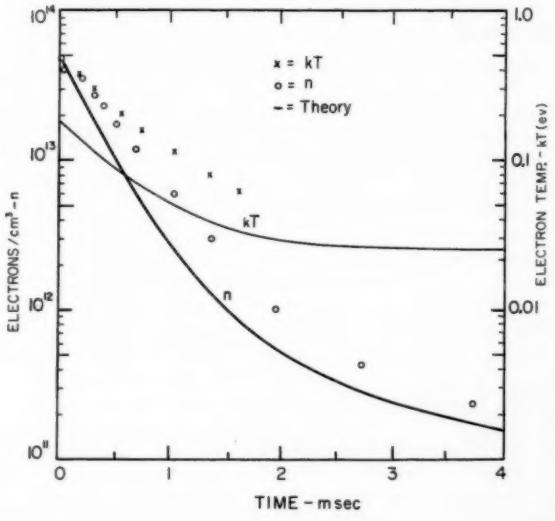

FIG. 2. Comparison of experimental (points) and theoretical (solid lines) time dependence of the electron density and electron temperature following discharges in helium. The helium pressure was 4 microns of mercury.

and in the absolute value of the three-body recombination coefficient give an estimated error of about a factor of three in the theoretical prediction of the electron decay rate. The theory predicts satisfactorily the time dependence of the electron temperature but not its absolute value. Whether this failure is related to the factors previously mentioned, to radial inhomogeneities in the plasma, or to the inapplicability of the standard conductivity-temperature relation is not known.

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ENERGY CONVERSION MECHANISM IN A BOUNDED MAGNETIZED CURRENT-CARRYING PLASMA*

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This Letter reports on an energy conversion mechanism in a bounded magnetized current-carrying plasma which supports the quasi-transverse electromagnetic waves and quasi-longitudinal space-charge waves of Hahn[1] and Ramo. These waves for a finite plasma are coupled. The coupling under certain circumstances can simulate a traveling-wave tube[2] type of interaction and thus can provide a means of transferring the kinetic energy of the drifting plasma to the electromagnetic energy, and vice versa. The essence of the analogy lies in the ability of the plasma to support slow electromagnetic waves. In a plasma composed of electrons and one kind of ion, analysis shows that for frequencies near electron or ion cyclotron frequencies the plasma indeed acts like a slow-wave structure.[3] In a drifting current-carrying plasma for frequencies in the vicinity of the ion cyclotron frequency, the ions will provide the slow-wave structure for circularly polarized transverse electromagnetic waves. This slow wave interacts with the longitudinal space-charge waves for electrons and thus grows at the expense of the drift energy of the electrons. The narrowness of the interaction region, which is indicative of the narrowness of velocity bandwidth for gain, forces one to choose the frequency of the electromagnetic wave close to and slightly below the ion cyclotron frequency. This choice consequently may permit the transfer of energy from the growing electromagnetic wave to the ions, and thus achieve plasma heating.[4] Thus, such a scheme, if efficient, can utilize the destructive energy of "runaway electrons" to heat the thermonuclear plasma. It is to be remembered that the critical temperature region for which the high-frequency plasma heating is efficient might be different from the temperature region for which runaway electrons appear; this scheme of energy conversion might act to prevent the instabilities due to runaway electrons even if not enhancing the plasma heating simultaneously. In addition, if such a traveling wave interaction between the streams of electrons and slow electromagnetic waves exists in the ionosphere, then it may explain certain types of very low frequency radio noise.

In order to understand this energy conversion mechanism quantitatively, a linear macroscopic harmonic analysis of the drifting plasma is carried out. For the sake of simplicity it is assumed that the circularly cylindrical plasma partially fills a perfectly conducting cylindrical waveguide and is subject to an axial static mag-

netic field B_0.

The electrodynamic analysis, which neglects collision and pressure gradient terms, provides the following dispersion relation when the drifting plasma almost fills the waveguide.[5]

$$\frac{\hat{k}_2}{\hat{k}_1}\frac{J_0(\hat{k}_1 a)}{J_1(\hat{k}_1 a)}\frac{J_1(\hat{k}_2 a)}{J_0(\hat{k}_2 a)} = \frac{k_2^2 - \hat{k}_2^2}{k_2^2 - \hat{k}_1^2}, \tag{1}$$

where $\hat{k}_1$ and $\hat{k}_2$ are the radial wave numbers for coupled E and H plasma-guide modes. When the coupling as displayed by the deviation $(k_2 - \hat{k}_2)$ vanishes, we get pure E and H plasma-guide modes, and $\hat{k}_2 \to k_2$ and $\hat{k}_1 \to k_1$. k_1 and k_2 are the radial wave numbers corresponding to the uncoupled E and H plasma-guide modes.

The k_1 and k_2 are complicated functions of plasma parameters. The preliminary analysis of this dispersion relation for proper choices of parameters indeed suggests the growth potentialities. The choice of the parameters is restricted in such a manner that the phase velocity of the quasi-transverse electromagnetic waves and the dc velocity of the electrons relative to ions in the plasma are almost the same.

The criterion for growth is

$$k_0^2\left(1 - \frac{\omega_{p+}^2}{\omega^2 - \omega_{H+}^2}\right) \geq \beta_0^2 + P \geq k_0^2\left(1 - \frac{\omega_{p-}^2}{\omega^2} - \frac{\omega_{p+}^2}{\omega^2 - \omega_{H+}^2}\right), \tag{2}$$

where

$$P = k_0^2\left\{1 - \frac{\beta_{p-}^2(\beta_0 - k)^2/\beta_0^2}{(\beta_0 - k)^2 - \beta_{H-}^2} - \frac{\omega_{p+}^2}{\omega^2 - \omega_{H+}^2}\right\} - k^2, \tag{3}$$

with $k_0 = \omega/c$, ω being the modulating frequency and c the velocity of light; k = axial wave number; $\beta_{p-} = \omega_{p-}/V_0$, $\beta_0 = \omega/V_0$, and $\beta_{H-} = \omega_{H-}/V_0$, V_0 being the drift velocity of electrons relative to the ions; ω_{H-} and ω_{H+} are the electron and ion cyclotron frequencies; ω_{p-} and ω_{p+} are the electron and ion plasma frequencies.

For the following two cases of physical interest the dispersion relation has complex roots.

Case 1. Consider a whistler-type signal propagating along the earth's magnetic field and interacting with a plasma streaming along the magnetic field. The appropriate parameters for this case are

$$f \approx 7.16\times10^2 \text{ cps}, \quad f_{p-} \approx 2.8\times10^7 \text{ cps},$$

$$f_{H+} \approx 7.63\times10^2 \text{ cps}, \quad f_{p+} \approx 6.6\times10^4 \text{ cps},$$

$$V_0 = 3\times10^7 \text{ meters/sec}, \quad N_+ = N_- = N,$$

$$B_0 = 0.48\times10^{-4} \text{ weber/meter}^2 \text{ (0.48 gauss)},$$

$$N = 10^{15}/\text{meter}^3.$$

For these parameters the wave number k is given by

$$k = (1.5\times10^{-4} \pm j\, 0.667\times10^{-6})M^{-1}. \tag{4}$$

It is to be remembered that this gain as shown by the imaginary part of the wave number corresponds to the case when the plasma is surrounded by perfectly conducting waveguide. For whistler mode propagation in the outer atmosphere, the walls of the guide need to recede to infinity. In this case the gain will be smaller than the one reported here in (4), and may still explain the very low frequency whistler noise.[6]

Case 2. Consider a laboratory plasma with the following parameters:

$$f \approx 4.5 \text{ Mc/sec}, \quad f_{p-} \approx 2.8\times10^3 \text{ Mc/sec},$$

$$f_{H+} \approx 4.6 \text{ Mc/sec}, \quad f_{p+} \approx 6.6 \text{ Mc/sec},$$

$$V_0 = 3\times10^7 \text{ meters/sec}, \quad N_+ = N_- = N,$$

$$B_0 = 0.34 \text{ weber/meter}^2 \text{ } (3.4\times10^3 \text{ gauss}),$$

$$N = 10^{17}/\text{meter}^3.$$

For these parameters the wave number k is given by

$$k = (0.9 \pm j\, 0.221)M^{-1}. \tag{5}$$

Pertinent information regarding the derivation of Eq. (1), the significance of the auxiliary parameter P, the choice of optimum experimental parameters for gain, the gain bandwidth, and the efficiency of the energy conversion mechanism will be discussed in a forthcoming paper. Likewise, as the interaction is mainly supported by the scalloping edge of the bounded plasma, the knowledge of the effects of pressure gradient terms on the gain bandwidth will also be discussed.

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HOT ELECTRONS IN METAL FILMS: INJECTION AND COLLECTION

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Studies of electronic phenomena in metals have been made wherein electrons having energies in the range between the Fermi energy and the vacuum level were injected into a metal by quantum tunneling through a thin insulating film. Mead[1] has reported tunnel emission of electrons into a thin metal film in this fashion, where between 0.1 and 0.3 of the emitted electrons were detected by a silicon monoxide-aluminum electrode adjacent to the metal film.

This Letter describes an experiment using a structure in which energetic electrons are injected into a metal, the structure affording an easy method of separating these hot electrons from those around the Fermi level. This experiment provides a convenient way to measure the properties of these hot electrons.

The source of injected electrons is a metal, separated from the base metal into which the electrons are injected through a thin oxide film (see Fig. 1). The electrons can pass through this film only by a quantum mechanical tunneling through the forbidden region. The oxide permits a potential difference V_{EB} to be established between the two metals, and so the electrons arrive in the base metal at an energy approximately qV_{EB} above its Fermi level. The base metal should be thin enough to assure that there is small probability of the electron losing any of this additional energy in a collision in the film. If this condition is met, the electron will arrive at the opposite face of the film qV_{EB} electron volts above the Fermi level. If a second barrier, of height $\phi_C < qV_{EB}$, exists at this face, it will separate the film from a "collector" region. The carrier will pass over this barrier into the collector, and contribute to the current, I_C, in the collector circuit. Lower energy electrons in the metal film, however, cannot surmount ϕ_C, and hence cannot contribute to I_C. Thus, I_C is dependent only upon properties of these hot electrons.

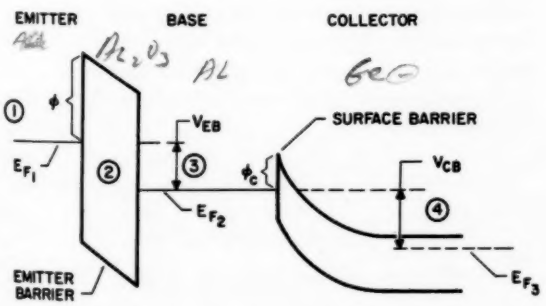

FIG. 1. Electron potential diagram of thin-film structure.

The structure was fabricated as follows: An aluminum film about 100 A thick was evaporated on the etched surface of a 1 ohm-cm, n-type piece of germanium. This created a collector surface barrier at the interface.[2] A thin oxide film was then formed on the surface of the aluminum film. A film of gold was evaporated over the oxide to serve as a source of electrons.

As V_{EB} is increased from zero, the quantum mechanical tunneling of electrons[3] from (1) to (3) occurs (see Fig. 1) and collection can be observed. Figure 2 is a plot at room temperature of I_C vs V_{CB}, and Fig. 3 shows I_C vs V_{EB} for seven different constant values of I_E. These data show low impedance between (1) and (3) indicating that tunneling is occurring, high impedance between (3) and (4) indicating that the

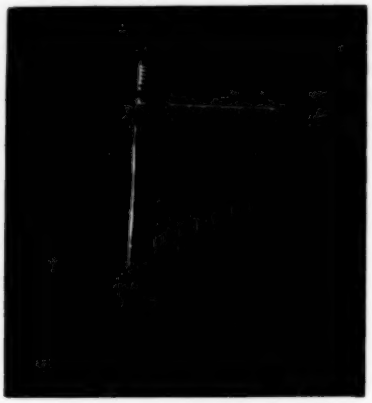

FIG. 2. I_C versus V_{CB} for various values of I_E. Vertical scale: 0.2 volt/division; horizontal scale: 0.5 milliampere/division. $\Delta I_E = 1$ milliampere/step; $T = 300°K$.

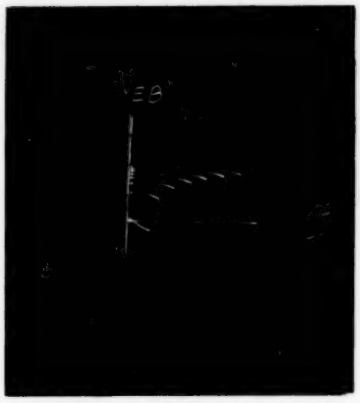

FIG. 3. I_C versus V_{EB} for various values of I_E. Vertical scale: 1 volt/division; horizontal scale: 0.5 milliampere/division. $\Delta I_E = 1$ milliampere/step; $T = 300°K$.

collector is insensitive to low-energy electrons, and a high collection efficiency. About 75% of injected electrons were collected in this experiment. In other experimental structures, up to 90% of the injected electrons have been collected. These data show that the structure can be utilized as an amplifier—similarly to a normal transistor. Furthermore, the geometry employed and the bias configuration preclude the possibility of this effect being due to normal bipolar or analog transistor operation.

Information on scattering processes in the metal film can be deduced from collection efficiency data. For example, if a single collision results in a loss of all the excess energy of an injected electron, then the number of electrons having an excess energy ΔE will vary with distance into the film as

$$n_{\Delta E}(x)/n_{\Delta E}(x_0) = e^{-(x-x_0)/\lambda},$$

where λ = mean free path between collisions.

The 90% collection efficiency quoted above thus implies a mean free path of at least 1000 A. This is significantly greater than the usual numbers quoted for mean free paths of electrons in metals, and suggests that the electrons may undergo many collisions with only a small percentage loss in their excess energy. Even backscattering need not degrade collection efficiency if followed by reflection at the metal-oxide interface. Therefore, the requirements for thicknesses of base metal are not nearly so stringent as one would anticipate from previous data on mean free paths.

Transfer characteristic curves, such as those shown in Fig. 3, also show evidence that reverse biases on the collector influence the current which can pass through the emitter. The effect cannot be explained on the basis of a voltage drop occurring across the lateral base resistance.

Data taken at lower temperatures ($\approx -100°C$) showed a threshold value of V_{EB} of ≈ 0.6 volt before any collection was achieved. This threshold can serve as a direct and accurate measure of the contact potential at metal-semiconductor interfaces.

We wish to thank C. V. Bocciarelli and R. A. Williams for encouragement and support, Dr. Ernest Pittelli for stimulating discussions, Walter Witt for important experimental contributions, and Louis J. Gringeri for fabricating the structures studied.

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ORIGIN OF THE F^{19} HYPERFINE STRUCTURE IN TRANSITION ELEMENT FLUORIDES*

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The F^{19} hyperfine structure has now been observed in many iron series fluorides[1] by both electron paramagnetic resonance[2] and nuclear magnetic resonance.[3] In a new theoretical investigation of the origin of this effect we have found that all previous phenomenological interpretations and theoretical analyses have neglected terms which are important for understanding the magnetic interactions and for determining the electron distributions in these materials.

The large internal magnetic fields that have been found to exist at the nucleus of normally diamagnetic ions, like F^-, have been interpreted as arising from unpaired electron spins in the fluoride ion orbitals. Since the origin of the unpairing lies in the interaction of these electrons with the $3d$ magnetic electrons of the iron series atoms, the experiments have been shown to yield valuable information about the distribution of the magnetic electrons and have greatly increased our understanding of the behavior of these materials. Very briefly, the theoretical interpretations of the hfs observations have followed several distinct models which describe the unpairing as being the result of (1) an admixture of covalent bonding into the purely ionic configuration[2,4] (i.e., the metal ion $3d$ wave functions have been augmented by small amounts of fluorine functions of the proper symmetry), or (2) the unpairing action of the Pauli principle (on the wholly ionic configuration) which affects those fluoride orbitals which have the same spin as the cation $3d$ orbital differently from those orbitals which have opposite spin.[5] In either method the unpaired fluorine s electrons produce an isotropic hfs (from the contact part of the Fermi interaction) whereas the $2p$ electrons are responsible for the anisotropic interaction. For lack of space we shall only discuss the isotropic shifts, but our findings are applicable to a lesser degree to the anisotropic interaction as well.

In the covalent bonding scheme [(1) above], the degree of unpairing is measured by the fraction f_S of $2s$ ligand wave function admixed into the $3d$ wave function and is given by $(S+\lambda)^2$, where S is the overlap integral between the $3d$ (metal ion) and the $2s$ (ligand) wave functions and λ is the covalent mixing parameter. {In the ionic model [(2) above], the mixing is determined by S^2 (plus several other small terms) and by definition the degree of covalency λ is zero.} If the measurements are interpreted on this basis, then the degree of covalency λ is determined and this important information about the electron densities has already been put to use in a variety of ways.[6]

Let us consider these theories in some detail now. For simplicity we follow the usual procedure and reduce the problem to a consideration of the diatomic system, say Mn^{++}-F^-. The antibonding molecular orbitals (i.e., the augmented Mn^{++} $3d$ wave functions) may be written as

$$\Phi = N[\psi_{3d}(Mn^{++}) - (S_{2s}+\lambda)\phi_{2s}(F^-)], \qquad (1)$$

where S_{2s} is the overlap integral

$$\int \psi_{3d}^*(Mn^{++})\phi_{2s}(F^-)dv,$$

and the normalization factor is

$$N = (1+\lambda^2 - S_{2s}^2)^{-1/2} \qquad (2)$$

if ψ_{3d} and ϕ_{2s} are normalized. (The ionic model gives a similar result, but with $\lambda = 0$, if one-electron functions in the Slater determinant are transformed to an orthogonal set.)

The unpaired, augmented $3d$ orbital interacts with the magnetic moment of the nucleus, via the Fermi contact term, to give an isotropic hfs which depends on the electron's spin density at the fluorine nucleus. In general this spin density is written as being due solely to the fractional $2s$ admixed into the wave function. It produces an effective field at the fluorine nucleus which

is 4π times this spin density, or

$$H_e = 4\pi N^2 (S_{2s} + \lambda)^2 |\phi_{2s}(0)|^2, \quad (3)$$

in atomic units (i.e., 1 a.u. $= 4.2 \times 10^4$ gauss). No other terms are included as these are thought to be of little importance. In particular the unpairing action of the 1s electrons is disregarded on the grounds that S_{1s} is much smaller than S_{2s} (and this is certainly true). On the other hand, $|\phi_{1s}(0)|^2$ is appreciably larger than $|\phi_{2s}(0)|^2$ and this physical fact has important consequences which have been overlooked.

Consider the augmented 3d function of Eq. (1) only augmented by $\phi_{1s}(F^-)$ and limit the covalency to the 2s electrons.

$$\Phi = (1 + \lambda^2 - S_{2s}^2 - S_{1s}^2)^{-1/2} \times [\psi_{3d}(Mn^{++}) - (S_{2s} + \lambda)\phi_{2s}(F^-) - S_{1s}\phi_{1s}(F^-)], \quad (4)$$

and the spin density at the F^- nucleus due to the 2s and 1s electrons gives an effective field

$$H_e = N'^2 \{(S_{2s} + \lambda)^2 |\phi_{2s}(0)|^2 + S_{1s}^2 |\phi_{1s}(0)|^2 + 2(S_{2s} + \lambda) S_{1s} \phi_{2s}(0)\phi_{1s}(0)\}. \quad (5)$$

(The same terms also occur in the ionic model, but with $\lambda = 0$.)

$KMnF_3$ will be used to indicate the magnitude of the effect of using Eq. (5) instead of the usual Eq. (3). Values for $4\pi|\phi_i(0)|^2$ and S_i (appropriate for an $F^- - Mn^{++}$ distance of 3.95 a.u.) appear in Table I. These were calculated using recently obtained Hartree-Fock functions for[7] Mn^{++} and for[8] F^- which we believe to be more exact Hartree-Fock functions than the ones used in previous investigations.[3-5] The 2s parameters used here are rather different from the values [$S_{2s} = 0.05$ and $4\pi|\phi_{2s}(0)|^2 = 145.2$ a.u.] used previously—an indication that such parametrizations will be sensitive to the wave functions used.

Shulman and Knox[9] have observed an H_e of +0.753 a.u. for $KMnF_3$. Setting $\lambda = 0$ and using the parameters of Table I, Eq. (3) predicts an H_e of 0.509 a.u. in fair agreement with experiment. The three terms of Eq. (5), however, are 0.509, 0.055, and -0.334 a.u., respectively, yielding an H_e of 0.230 a.u. in rather poor agreement with experiment. This difference is due to the large negative cross term—a term which is always of opposite sign to the other terms because ϕ_{2s} has a node and ϕ_{1s} does not. We could instead take the observed H_e of 0.753 and solve the equations for λ. Doing this, one obtains λ's of 0.013 from Eq. (3) and 0.033 from Eq. (5). Inclusion of the unpairing of the 1s electrons has altered the estimate of covalency by a factor of three, while it has increased the value of $(S + \lambda)$ by 27%.

Table I. $4\pi|\phi_i(0)|^2$ for F^- and overlap integrals, S_i, appropriate for an $F^- - Mn^{++}$ distance of 3.95 a.u.

$S_{1s} = 0.00453$	$4\pi\|\phi_{1s}(0)\|^2 = 2672.6$
$S_{2s} = 0.06166$	$4\pi\|\phi_{2s}(0)\|^2 = 136.58$

Further, it is clear from Eq. (4) that there are three other terms which contribute to the spin density and hence an H_e at the F^- nucleus (these terms also appear in the ionic model via a different mechanism). While two of these have been discussed,[5] they have, in general, not been included into covalent analyses of experiments. These terms are $|\psi_{3d}(r_F)|^2$, $-2(S_{2s} + \lambda)\psi_{3d}(r_F) \times \phi_{2s}(0)$, and $-2S_{1s}\psi_{3d}(r_F)\phi_{1s}(0)$, where $\psi_{3d}(r_F)$ is the magnitude of the Mn^{++} free atom 3d function evaluated at the F^- nucleus. The first of these is 0.0005 a.u., the second +0.104 a.u. (with $\lambda = 0$), and the last -0.035 a.u. Their sum, 0.069 a.u., is indeed not negligible since it is 14% of the result using Eq. (3) and 30% of the result using Eq. (5).

Thus, a proper treatment of the problem, either phenomenologically or theoretically, requires the simultaneous inclusion of all the terms discussed in this Letter. These matters, the role of exchange polarization, and the expansion of the cation (and the contraction of the anion) wave functions will be reported on in a future paper.

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†Guests of the Solid-State and Molecular Theory Group, Massachusetts Institute of Technology, Cambridge, Massachusetts.

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DIRECTION OF INTERNAL MAGNETIC FIELDS IN RARE EARTH IONS*

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It has recently been reported[1,2] that the effective magnetic field acting at the nuclei of Fe atoms in ferromagnetic iron has a direction opposite to that expected for the uncompensated spins of the electrons in the 3*d* shell. It has been suggested[3] that this fact may be explained by taking into account the exchange interaction between the *d* and inner core *s* electrons. This interaction leads to a difference in the wave functions of the *s* electrons which have their spins parallel or antiparallel to the direction of the uncompensated *d*-electron spins. In Fe this results in a contribution to the magnetic field at the nucleus opposite to that expected without this effect and presumably larger than all other contributions to the internal field.

It is the purpose of this note to report on the signs of magnetic fields acting at the nuclei of rare earth ions in rare earth iron garnets.[4] We have used the technique previously described,[5] of measuring the rotation of the angular correlation pattern obtained in $1^+ \rightarrow 2^+ \rightarrow 0^+$ transitions in rare earth nuclei in magnetic materials. This rotation is measured relative to the direction of a weak magnetizing field applied to the garnet samples perpendicular to the plane of the two gamma-ray detectors used to measure the angular correlation pattern. Because the *g* factors of 2^+ rotational states are known to be positive, the sign of the internal field is given uniquely by the sense of rotation. We have measured the sense of rotation for $2^+ \rightarrow 0^+$ transitions in Sm^{152}, Gd^{154}, and Dy^{160} following the decay of Eu^{152}, Eu^{154}, and Tb^{160}, respectively, in polycrystalline samples of rare earth iron garnets. We find positive rotations for Sm^{152} and Gd^{154} and a negative rotation for Dy^{160}, where a positive rotation is defined to be one which would be obtained if the nucleus were precessing in the applied magnetizing field alone. These measurements were carried out at room temperature.

To interpret these results we make use of the following: (1) The direction of magnetization of the sample is given by that of the uncompensated spins, $\vec{S}_{Fe}$, of the iron sublattices. (This is true for Eu and Sm iron garnet over the whole temperature range and for Tb and Gd iron garnets above their compensation temperatures, e.g., at room temperature.[6] (2) The orbital angular momentum $\vec{L}$ of the rare earth ions is not quenched[6] in the garnets at room temperature and the relative orientation of $\vec{L}$ and $\vec{S}$ is given by Hund's rule. (3) The spins, $\vec{S}$, of the rare earth ions are antiferromagnetically oriented with respect to the uncompensated spins, $\vec{S}_{Fe}$, of the iron sublattices.

Dy^{160}: In both the parent $Tb(^7F_6)$ and the daughter $Dy(^6H_{15})$, Hund's rule requires that $\vec{L}$ and hence $\vec{J}$ are parallel to $\vec{S}$. Thus, $\vec{J}$ is oriented opposite to $\vec{S}_{Fe}$ and hence opposite to the direction it alone would possess in the applied magnetizing field. Hence, we expect a negative rotation if the internal magnetic field is produced mainly by the *f*-shell electrons in the rare earth ion. This prediction agrees with experiment. [To check our picture we have also measured the sign of rotation of the Dy^{160} below the compensation temperature of the host Tb crystal. In this case we predict and find a positive rotation.]

Sm^{152}: Because $Sm(^6H_{5/2})$ has less than a half-filled shell, Hund's rule requires that $\vec{L}$ be antiparallel to $\vec{S}$ and hence parallel to $\vec{S}_{Fe}$. Since $|\vec{L}| > |\vec{S}|$, $\vec{J}$ is oriented in the direction of $\vec{S}_{Fe}$, and we expect a positive rotation. This predic-

tion agrees with experiment. (We assume here that the atomic state existing after K capture from Eu^{152} is that of Sm^{152}. Evidence for this assumption will appear in a study of relaxation times and internal fields in rare earth iron garnets which is in preparation for publication.)

Gd^{154}: Both the parent $Eu^{154}(^7F_0)$ and daughter $Gd^{154}(^8S_{7/2})$ have ground-state configurations that to first order produce vanishing internal magnetic fields. This is in agreement with the fact that the rotations in this case correspond to average fields of 50 kilogauss, while it is of the order of megagauss in Sm and Dy at the same (room) temperature. If one tries to account for this small residual rotation by taking into account higher multiplets, one finds it difficult to explain the observed sign of the rotation. In particular, if one assumes that the low-lying $J=1$ state in Eu (which contributes to the magnetic moment of the Eu ion) is responsible for the hyperfine interaction, a negative rotation is predicted, in disagreement with experiment.

It is clear that in Sm^{152} and Dy^{160}, unlike the case in Fe, the contributions to the internal magnetic field arising from s electrons exchange-coupled to the f-shell electrons are not sufficiently large to change the sign of the observed internal field. In the case of Eu or Gd, however, the main source of field may be the exchange-coupled contribution. If the exchange-coupled contribution does not vary appreciably among the various rare earths, this interpretation is consistent with the results obtained in Sm and Dy. With the further assumption that the exchange-coupled interaction is of the form $A'\vec{I}\cdot\vec{S}$, the sign of A' for Eu or Gd is determined to be negative.

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MAGNETIC FIELD DEPENDENCE OF THE SUPERCONDUCTING ENERGY GAP

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Gor'kov[1] has recently derived the Ginzburg-Landau[2] (GL) phenomenological equations of superconductivity from the microscopic theory in the region where $T_c - T \ll T_c$ and in the London or local limit where the penetration depth $\lambda(T)$ is much greater than the coherence length ξ. His main point was that the effective charge, e^*, in the GL theory was equal to twice the electronic charge; but perhaps of even greater significance is the fact that the order parameter ψ of the GL theory is proportional to the energy gap. Bardeen, Cooper, and Schrieffer[3] (BCS) had previously suggested that the energy gap could be used as an order parameter for nonlinear extensions of their theory.

The success of the nonlinear GL theory has been its ability to explain the behavior of superconductors in strong magnetic fields better than any of the linear theories such as that of London and London, to which it reduces for weak fields. Inasmuch as the BCS theory is capable of handling the application of a magnetic field only as a perturbation, one would expect the solution of their microscopic equations for strong fields to be quite difficult. A weak-field calculation of the field dependence of the energy gap has been done by Gupta and Mathur.[4] On the other hand, the GL equations are relatively easy to solve for strong fields, and in the light of Gor'kov's result the solutions would be expected to have the same rigor as those of the microscopic theory. The restriction of locality, $\lambda \gg \xi$, is perhaps not a serious one, for there is evidence[5,6] that when this condition is not satisfied one may incorporate the nonlocal effects into the local theory by letting the penetration depth be a func-

tion of ξ and the mean free path. If one is considering thin films which are prepared by evaporation, then one would expect the coherence distance to be limited by the size of the crystallites. The locality condition would then be $\lambda \gg d$. Thus any superconductor prepared in the form of a very thin evaporated film will satisfy this condition. In regard to the restriction $T_c - T \ll T_c$, Ginzburg[7] has suggested that the order parameter is small even far below T_c. Thus the theory should also be valid at all temperatures below T_c. Bardeen[8] and Ginzburg[7] have considered forms for the full temperature dependence of the equations. However, the question of removing these restrictions is still a moot point, and the application of the equations derived here is only rigorous in the local limit and near T_c.

In the GL theory, ψ is a function of temperature, magnetic field, and coordinates; therefore, so is the energy gap. The question of finding the field dependence of the energy gap reduces to solving the GL equations for ψ.

Consider a plate of thickness d with an external field H_0 applied parallel to the surface. The GL equations have already been solved for this case[9,10] mainly in connection with calculations of the critical field. If $d/\lambda(T) \ll 1/\kappa \approx 10$, where κ is the nonlinear coupling constant of the theory, ψ is independent of coordinates and the two independent equations of condition found by them are

$$H_0^2 = \frac{4\phi_0^2(\phi_0^2 - 1)\cosh^2[\phi_0 d/2\lambda(T)]}{1 - [\lambda(T)/\phi_0 d]\sinh[\phi_0 d/\lambda(T)]} H_{cb}^2, \quad (1)$$

$$H_c^2 = \frac{\phi_c^2(2 - \phi_c^2)}{1 - [2\lambda(T)/\phi_c d]\tanh[\phi_c d/2\lambda(T)]} H_{cb}^2, \quad (2)$$

where $\phi = \psi(T,H)/\psi(T,0)$, $\lambda(T)$ is the temperature-dependent London penetration depth (or perhaps an effective penetration depth), $H_{cb}(T)$ is the bulk critical field, ϕ_0 is the equilibrium value of ϕ in the presence of H_0, and ϕ_c and H_c are the critical values. Gor'kov's result can be stated as

$$\epsilon(T,H)/\epsilon(T,0) = \psi(T,H)/\psi(T,0) = \phi, \quad (3)$$

where $\epsilon(T,H)$ is the energy gap. Equation (1) describes a smooth decrease in the energy gap as the field is increased. As $H_0 \to H_c$ the gap approaches a critical value. For this critical gap, the superconductor is in equilibrium with the normal state. Setting (1) equal to (2) gives ϕ_c, from which one can obtain H_c.

For $d/\lambda(T) \ll 1$, the solutions of Eqs. (1) and (2) are

$$\epsilon(T,H_c)/\epsilon(T,0) = 0, \quad (4)$$

$$H_c^2 = 24[\lambda(T)/d]^2 H_{cb}^2(T), \quad (5)$$

$$\left[\frac{\epsilon(T,H_0)}{\epsilon(T,0)}\right]^2 = 1 - \frac{1}{24}\left(\frac{d}{\lambda(T)}\right)^2\left(\frac{H_0}{H_{cb}(T)}\right)^2, \quad (6)$$

or, using (5),

$$\left[\frac{\epsilon(T,H_0)}{\epsilon(T,0)}\right]^2 = 1 - \left(\frac{H_0}{H_c(T)}\right)^2. \quad (7)$$

Thus for very thin films the critical gap is zero, and the field dependence of the gap is independent of thickness when the external field is normalized to the critical field of the film. A closer examination of Eqs. (1) and (2) shows that the critical gap remains zero all the way up to $d/\lambda(T) = \sqrt{5}$. Therefore, the superconducting phase transition should be second order for films thinner than this value. This result was previously given by GL.

For small H_0, Eqs. (1) and (2) can be solved to give

$$\frac{\epsilon(T,H_0)}{\epsilon(T,0)} = 1 - \tfrac{1}{6}\left[\frac{\lambda(T)/d]\sinh[d/\lambda(T)] - 1}{\cosh^2[d/2\lambda(T)]}\right]\left(\frac{H_0}{H_{cb}(T)}\right)^2. \quad (8)$$

This expression is good for all d.[11]

The value of the critical gap has been calculated as a function of $d/\lambda(T)$ and is plotted in Fig. 1.

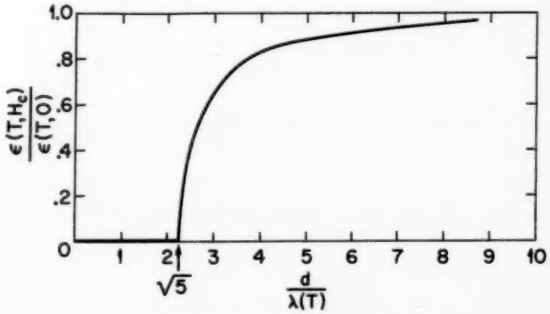

FIG. 1. Energy gap at the critical field vs thickness.

Ginsberg and Tinkham[12] and Richards and Tinkham[13] have looked for a change in the energy gap with field in thin films and in bulk metal, respectively, using infrared techniques. No change in the gap was observed on a 12A film of lead in a field of 8000 gauss. Application of Eq. (6) shows that the change in the gap under these conditions is less than 1%, a change too small for them to measure. Also, no change in the energy gap for bulk superconductors was observable. From Fig. 1 it is seen that the maximum change in the gap approaches zero as the thickness approaches that of bulk metal. Although the expressions derived here are not valid for $d/\lambda(T) > 10$, GL have shown that for bulk superconductors the maximum change in the order parameter (energy gap) with field is of the order of a few percent.

Glaever and Megerle[14] have recently made direct measurements of the energy gap of aluminum as a function of magnetic field. Their results are shown in Fig. 2. Calculating the penetration depth as suggested by Tinkham,[5] one obtains for an Al film of thickness 1600 A at $T = 1.05°K$ a value of 2460 A for λ. This makes $d/\lambda \approx 0.65$. Thus from Fig. 1 the transition should be second order and the energy gap should go smoothly to zero. The data indicate that this is true. The field dependence of the energy gap as predicted by Eq. (7) is also shown in Fig. 2, and it is seen that the experimental points all lie below the theoretical curve. Perhaps the reason for the disagreement is that the theoretical model of an ideal homogenous film is too simple and one must consider that the film is really made of small crystallites in order to bring the theory into closer agreement with experiment.

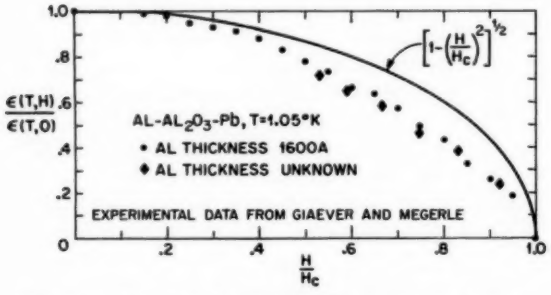

FIG. 2. Energy gap of aluminum vs magnetic field.

The results of the Gupta-Mathur calculation show that for thin samples the gap is always finite in the superconducting state and that the maximum change in the gap becomes smaller as the thickness becomes smaller, which disagrees with the results of this paper. The reason for this divergence is uncertain. Their calculation was done for the Pippard limit, $\xi \gg \lambda$, whereas ours was done for the London limit, $\xi \ll \lambda$; but it is difficult to envision that these different limiting cases would lead to a different thickness dependence. Also it was noticed that in the application of their results, Gupta and Mathur apparently used the bulk critical field instead of allowing the critical field to be a function of thickness; but even with this correction the two calculations are still in qualitative disagreement.

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NUCLEAR MAGNETIC SPIN-LATTICE RELAXATION BY SPIN-ROTATIONAL INTERACTIONS*

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The possible importance, in nuclear magnetic spin-lattice relaxation, of the magnetic fields produced by molecular rotation was pointed out by Bloembergen, Purcell, and Pound[1] in their classic paper on nuclear relaxation. The purpose of this Letter is to present experimental evidence that spin-rotational interactions provide the dominant relaxation mechanism in certain molecular liquids.

Earlier work[2] on the proton and fluorine T_1's at room temperature in CH_2FCl, CHF_2Cl, $CHFCl_2$, and 1, 3, 5-trifluorobenzene demonstrated that the anisotropy in the fluorine nuclear magnetic shielding contributes an appreciable, field-dependent, term to the fluorine T_1. In these molecules, the direct dipole-dipole interactions among the nuclei are nearly identical for protons and fluorine. Moreover, because of their rigid molecular framework, the molecules should have the same correlation time τ_C governing the T_1 of fluorine nuclei as well as of protons. Therefore, even though it is difficult to make a sufficiently accurate estimate of τ_C to predict quantitative values for T_1, the ratio of the T_1's, $R = T_{1H}/T_{1F}$, should be predictable to better accuracy. However, it was found that although the predicted ratio of the dipolar contributions to T_1, R_{dip}, ranged between 0.4 and 1.0 for the four liquids, the observed ratio ranged between 1.5 and 9.2. In addition, R_{obs} increased with the applied magnetic field, indicating the presence of a field-dependent interaction which decreased the fluorine T_1 but had little or no effect upon the proton T_1.

This behavior agrees qualitatively with the effects calculated on the basis of anisotropy $\Delta\sigma$ in the chemical shift tensor σ inasmuch as the proton shifts are too small for their anisotropy to contribute much to T_1. However, even though the anisotropy is large enough to be important for fluorine, estimates of its magnitude based on chemical shift theory and also on the field dependence of R_{obs} led to the result that R_{obs} should be about 1.5 R_{dip} for T_1's at 20 Mc/sec. Therefore, it was concluded that some additional more important factor was shortening the fluorine T_1 and causing the several-fold difference[1] between R_{dip} and R_{obs}.

That this additional relaxation mechanism involves the spin-rotational interaction is shown by our subsequent measurements of the temperature dependence of the proton and fluorine T_1's. Results for CH_2FCl, CHF_2Cl, and CHF_3 are presented in Fig. 1 as a semilogarithmic plot of T_1 versus $10^3/T°K$; qualitatively similar but less extensive data have been obtained[3] for $CHFCl_2$. The increase of T_1 with temperature implies that $\omega^2\tau_C^2 \ll 1$; however, the curvature of the plots for both the proton and fluorine T_1 from straight lines at higher temperatures indicates deviations from "ideal" behavior.[1] Taken individually, such deviations for either the proton or the fluorine T_1 could be attributed to any of several aspects of the liquid-state structure or purity. The significant feature is that the deviations are larger for fluorine than for the proton; and, moreover, the differential effect is itself strongly temperature dependent. This

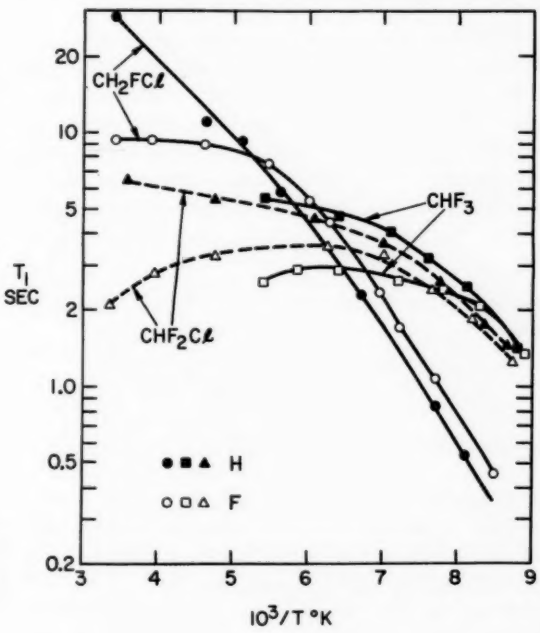

FIG. 1. The temperature dependence of the proton and fluorine T_1 observed by rf pulse methods at 20 Mc/sec in several liquid fluoromethanes.

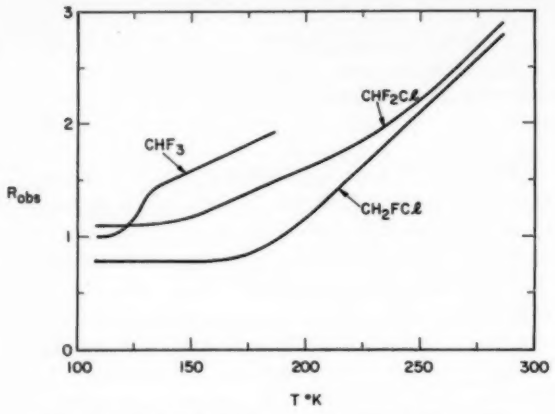

FIG. 2. The differential dependence upon temperature of the proton and fluorine T_1 in the same molecule. R_{obs} is defined as T_{1H}/T_{1F}, where the T_1's are taken from the experimental curves in Fig. 1.

may be seen in Fig. 2 when the data in Fig. 1 are replotted as R_{obs} versus T.

At low temperatures, R_{obs} approaches a constant, limiting value which is close to $1.5R_{dip}$. In other words, the proton T_1 at low temperatures is that given by nuclear dipole-dipole interactions alone, while that for fluorine is determined by the nuclear dipole-dipole interaction plus $\Delta\sigma$. At higher temperatures R_{obs} increases, and after an intermediate curved region it becomes a linear function of the temperature. This is equivalent to stating that at higher temperatures there is an additional contribution to the fluorine T_1 such that

$$1/T_{1F} \sim \tau_c(A + BT). \qquad (1)$$

The only plausible mechanism for these results appears to be the spin-rotation interaction.

First of all, the spin-rotation interaction should be small for the proton and much larger for fluorine. This is implicit in the proportionality which Ramsey[4] has pointed out between the spin-rotation constant and the second-order paramagnetic term in the nuclear magnetic shielding, inasmuch as the latter is small for protons[4] and large for fluorine.[2] It is shown more directly by the recent molecular beam measurements of the spin-rotation interaction constant for the proton and fluorine nuclei in HF, which give $|c_p| = 71$ kc/sec and $|c_F| = 305$ kc/sec.[5] The contribution to $1/T_1$ goes as the square of the interaction constant,[1] so the relaxation of protons should be only 1/10 to 1/100 as effective as for fluorine.[6]

Secondly, although the temperature dependence of spin relaxation via the spin-rotation interaction is difficult to calculate in detail, the observed limits are compatible with theory. The interaction, in general, is of the form $\vec{\mu}\cdot c\cdot\vec{J}$, where $\vec{\mu}$ is the nuclear magnetic moment, c is the spin-rotation interaction dyadic, and $\vec{J}$ the molecular angular momentum. Spin-lattice relaxation occurs because molecular collisions cause m_J and also J to change, thereby producing fluctuations in the rotational magnetic field at the nuclei. If, for simplicity, we consider a linear molecule and assume that all the relaxation processes are governed by the same correlation time, the fluorine T_1 for $\omega^2\tau_c^2 \ll 1$ is given by an expression of the form,[1,7]

$$1/T_{1F} = \tau_c[A + Cc_F^2\langle J(J+1)\rangle]. \qquad (2)$$

The first term includes the nuclear dipole-dipole and $\Delta\sigma$ contributions and the second, the spin-rotation effects. However, $\langle J(J+1)\rangle$ is proportional to the rotational energy which approaches RT at high temperatures and zero at low.

This agrees with the high- and low-temperature limits in R_{obs}, except that Eq. (2) predicts the same $T=0$ intercept for the high- and low-temperature straight-line portions of the curves in Fig. 2. The origin of the discrepancy may be seen by comparing the relative magnitudes in Eq. (2) of the mean square dipolar field A, at a fluorine nucleus, and the mean square rotational field, $Cc_F\langle J(J+1)\rangle$. The latter should be of the order of 10 to 100 times the former, even at 100°K, and R should also be 10 to 100 instead of ≈ 1 as observed. Therefore, it appears that the spin-rotation contribution to $1/T_1$ is "quenched" by intermolecular interactions at lower temperatures and that the quenching becomes less important at higher temperatures, which seems reasonable. In other words, the assumption in Eq. (2) that the nuclear relaxation is governed by a single correlation time for all processes is much too drastic. Allowance must be made for relaxation among rotational states and, in fact, an interesting feature of these results is the possibility of studying such processes by means of nuclear T_1 and T_2 observations. Work along these lines is in progress.

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GENERAL ANALYSIS OF OPTICAL, INFRARED, AND MICROWAVE MASER OSCILLATOR EMISSION*

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(Received February 13, 1961)

In this paper we will generalize the equations governing coherent emission from quantum mechanical amplifiers (microwave, infrared, and optical masers) using either electric or magnetic dipole transitions. Earlier analyses by Gordon, Zeiger, and Townes,[1] Feynman, Vernon, and Hellworth,[2] and Wang and Singer,[3] are incorporated in our generalization. The principal new result of our analysis is that we find that an amplitude modulation of the output is to be expected from all maser oscillators excepting those in which excited atoms are supplied at a notably greater rate than the depopulation rate due to coherent induced emission. This analysis also suggests that energy population inversions of electric dipole moments can be performed using coherent light sources in the same manner as with magnetic dipole moments. It appears, therefore, that modulation of an optical or infrared maser (e.g., for communication purposes) may be easily accomplished by varying the population of excited states through control of the pumping energy. This modulation may be both by absorption and by amplification. Amplitude modulation of maser oscillators has been observed in this laboratory[3] and elsewhere[4-6] and we will explain these observations.

The total density of active atoms is composed of n_2, the density of upper energy state molecules, times the relative quantum (statistical) weights g_1/g_2, and n_1, the density of lower energy state atoms. We define the difference as the effective number of molecules given by

$$N_e \equiv n_2(g_1/g_2) - n_1. \tag{1}$$

N_e is dependent on the pumping system. For nondegenerate levels, $g_1/g_2 = 1$. The superposition state of a molecule may be described by

$$\psi = a_1(t)u_1 + a_2(t)u_2, \tag{2}$$

where $|a_1(t)|^2$ is the time-dependent probability coefficient of the lower state and $|a_2(t)|^2$ is the time-dependent coefficient for the upper state. The two states are orthogonal and may be normalized, but the existence of other states must be considered in such a normalization procedure. The frequency of transition is given by

$$\omega_0 = (W_2 - W_1)h, \tag{3}$$

where W_2 is the energy of the upper energy level and W_1 is the energy of the lower level of the maser. The starting population distribution of the maser is assumed to be an inverted Boltzmann distribution (often referred to as a negative temperature). The inversion is assumed to satisfy the requirement that the molecular radiation rate will exceed the rate of energy dissipation (losses) so that a sustained oscillation can occur. After a given population distribution is established, the coherent induced radiation rate of the excited molecules, neglecting relaxation, collision processes, and spontaneous emission, is given by

$$\frac{d}{dt}[\hbar\omega_0 N_e |a_2(t)|^2]. \tag{4}$$

The radiation wavelength is, in general, much greater than the distance between molecules. In fact, it is physically realistic to assume that, for any maser system, the radiation field acts upon the entire molecular system. The molecular system is the entire collection of maser molecules.

The value of $|a_1(t)|^2$ and $|a_2(t)|^2$ can be determined by a perturbation calculation.[7] The interaction energy between an oscillating electric field E_1 and an effective electric dipole moment p is given by Eq. (5). (Alternatively, a magnetic dipole μ and oscillating magnetic field H_1 may be substituted.)

$$H' = E_1 \cos\omega t \int u_1^* P u_2 dv = pE_1 \cos\omega t. \tag{5}$$

P is the dipole moment operator of the pertinent electric or magnetic dipole transition, ω is the angular frequency of the radiation, and p is the matrix element between the two states along the direction of E_1 (suitably averaged, e.g., see Shimoda, Wang, and Townes[8]). The values of a_1 and a_2 are then obtained (assuming no deviation from resonance) $[(\omega - \omega_0) = 0]$ as

$$|a_1(t)|^2 = \sin^2\left(\int_0^t \frac{pE_1(t)}{\hbar}\,dt\right),$$

$$|a_2(t)|^2 = \cos^2\left(\int_0^t \frac{pE_1(t)}{\hbar}\,dt\right), \tag{6}$$

where the integral of $E_1(t)$ over time is needed to account for the variation of $|E_1|$ with the emission. For masers using magnetic dipole transitions, $\Delta m = \pm 1$, and $H' = -\frac{1}{2}(\mu_x \pm \mu_y)(H_x \mp iH_y) \times \cos\omega t$. Then in Eq. (6) the term $pE_1(t)$ should be replaced by $\mu H_1(t)$. Note that $\mu/\hbar$ is generally defined as γ (the magnetogyric factor) in the magnetic case. Hence, $p/\hbar$ may also be defined as γ for electric dipole transitions. By substituting (6) into (4) we obtain the rate of coherent induced emission and absorption.

Next consider the equation governing the growth and decay of the radiation field. The change in stored energy over the bandwidth ($\Delta\nu$) and the cavity loss rate, in a cavity of quality factor Q, is described by

$$\frac{d}{dt}\left[\frac{\epsilon E_1^2}{4\pi}\right] + \frac{\omega_0 \epsilon E_1^2}{2Q \times 4\pi}. \tag{7}$$

We next set this equal to the molecular energy emission rate. It is convenient to utilize a filling factor[9] F to account for the variation in density and dielectric constant (or permeability) between the radiation field volume and the volume of the active molecules. The equation governing the exchange of energy between molecules and the radiation is

$$\frac{d}{dt}(E_1^2) + \frac{\omega_0 E_1^2}{2Q} = 4\pi N_e F h\nu_0 \frac{d}{dt}\left[\cos^2\left(\int_0^t \frac{pE_1(t)}{\hbar}\,dt\right)\right]. \tag{8}$$

By using (6) for the value of a_2, we have a nonlinear differential equation which can be solved approximately or numerically. If N_e is assumed to be constant, Eq. (8) describes a maser which is excited and then permitted to radiate such as an ammonia maser, a two-level paramagnetic maser, an optical ruby or samarium maser when excited by pulsed light, etc. If we wish to describe a maser which is continuously pumped,[10] additional terms are added, and the right-hand side of (8) becomes

$$4\pi F\hbar\omega_0\left[N_e + \int R_e\,dt\right]\frac{d}{dt}(|a_2|^2), \tag{9}$$

where R_e is the rate of pumping to decrease the density of lower state molecules relative to the density of upper state molecules.

The solution of (8) gives the line shape of the energy of the emitted radiation. The amplitude and period of the amplitude modulations of a maser are related to the coherence and intensity of the emitted radiation. We have carried out numerical solutions using an electronic computer

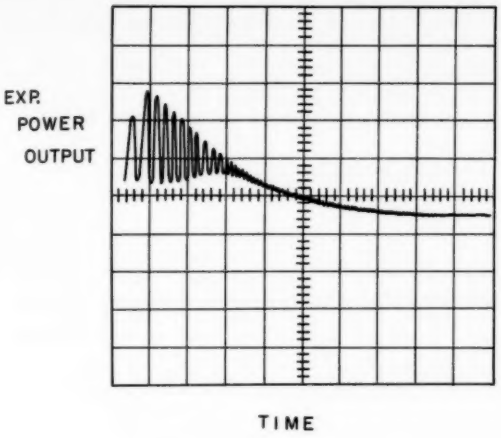

FIG. 1. Experimentally observed radiation envelope from a neutron-radiation-damaged calcite sample at *X* band [J. C. Kemp, Ph.D. dissertation, Electrical Engineering Department, University of California; issued as Electronics Research Laboratory Report No. 275, 1960 (unpublished)]. Activation is by a microwave adiabatic fast passage.

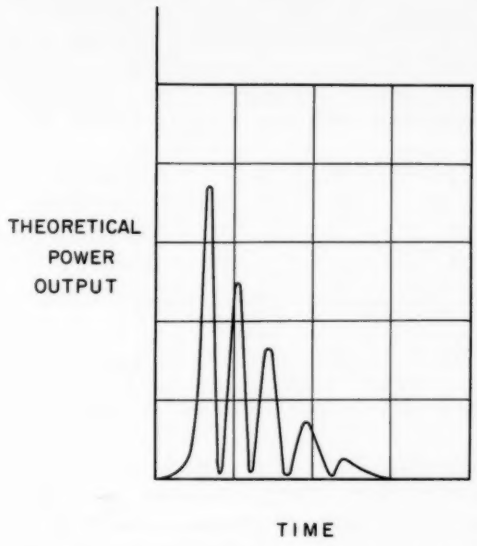

FIG. 2. Digital computer solution to Eq. (8). The scale is not the same as in Fig. 1. The amplitude and periods of amplitude modulation agrees with the experimental results within the experimental error.

for the case of a paramagnetic maser. The results agree with experiments as illustrated by Figs. 1 and 2.

An approximate result of the solution of (8) is that the output pulse will exponentially decay and will have amplitude modulation of a frequency

$$\omega_1 \sim (p/\hbar)(FN_e h\nu_0)^{1/2} \sim (p/\hbar)E_1. \qquad (10)$$

Since E_1 decays exponentially with time, the frequency of amplitude modulation will be higher at the peak amplitude than at lower output amplitudes. This has been experimentally observed. It may be noted from (10) that electric dipole two-level population inversion optical masers may be designed by supplying a pulse of radiation for a time given by $T=h/(2pE_1)$, where E_1 is the field strength of the coherent pulse of radiation. (This inversion is analogous to a paramagnetic spin echo first observed by Hahn.[11])

Amplitude modulation of the ammonia beam maser[1] is not prominent for the reason suggested in the first paragraph. A molecule passes through the maser cavity in an average time of about 10^{-4} second. Due to the low radiation energy density in the cavity, the period of amplitude modulation is greater than 10^{-4} second. Hence, the supply of upper state molecules exceeds the rate of loss by coherent radiation. In calculating the power output of the ammonia maser, Gordon, Zeiger, and Townes[1] were able to use an approximation which eliminated the nonlinear terms in our Eq. (9). The linearization is correct as noted in their paper for low-power masers such as the ammonia maser.

A more detailed analysis of the generalized equations, taking account of the molecular linewidth, is in the process of solution and will be published.

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*This work was supported in part by the Office of Scientific Research, by the Miller Institute for Basic Research in Science, and by the Faculty Research Committee.

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POLARIZATION AND THE TRIPLET ELECTRON-HYDROGEN SCATTERING LENGTH

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Recently, upper bounds for the triplet and singlet scattering lengths of electrons scattered from atomic hydrogen have been calculated by Rosenberg, Spruch, and O'Malley.[1] In the triplet case the upper bound was found to be $a_t \leq 1.91$.[2] This bound was obtained from an expression involving an arbitrary wave function ψ having a correct asymptotic form of an s-scattered wave at infinity. The particular function used by Rosenberg et al. did not contain a slowly vanishing part ($\sim -r^{-2}$) coming from the induced polarization in the hydrogen atom. A recent calculation[3] in which such a term was included gave a scattering length practically identical with the RSO bound, from which it may have been inferred that the polarization has an almost negligible effect on the scattering length and that the value $a_t = 1.91$ is very close to the correct scattering length. We wish to show that the polarization has a very important effect on the scattering length (of the order of 10%). Secondly, we want to report an accurate calculation of the triplet scattering length:

$$a_t = 1.74, \tag{1}$$

where the error is estimated to be less than three percent. The difference between this value and that of RSO is almost wholly due to the long-range polarization.

The effect of polarization as well as the calculation may be understood from a method which has been outlined in this journal.[4] The basic formula of that approach is

$$\sin(\delta - \delta_0) = \frac{-1}{k} \sum_{l=1}^{\infty} \frac{2}{(2l+1)^{1/2}} \int_0^{\infty} dr_1 \int_0^{r_1} dr_2 \Phi_0^{(0)} \frac{r_2^{\,l}}{r_1^{\,l+1}} \Phi_l. \tag{I.5}$$

δ is the exact phase shift of the whole problem and $\Phi_l(r_1 r_2)$ are "coefficients" in the expansion of the exact s-wave wave function in terms of Legendre polynomials $P(\cos\theta_{12})$, Eq. (I.1). δ_0 is the phase shift associated with a zeroth order problem, Eq. (I.4), which in particular does not contain any induced multipole distortion (polarization) effects.

Assume that we have computed δ_0 and all $l > 1$ terms on the right-hand side of (I.5). Assume also that only that part of the $l = 1$ term has been calculated by integrating up to a point R beyond which the difference between Φ_1 and its asymptotic form,

$$\lim_{r_1 \to \infty} \Phi_1 = \frac{-2}{\sqrt{3}} \frac{\sin(kr_1 + \delta)}{r_1^{\,2}} e^{-r_2(\frac{1}{2}r_2^{\,3} + r_2^{\,2})}, \tag{I.6}$$

can be considered negligible. Let $\delta(R)$ be the phase shift so calculated, and let $a(R)$ be the associated scattering length:

$$\lim_{k \to 0} \delta(R) = \pi - a(R)k.$$

$a \equiv a(\infty)$ is the exact scattering length. We find

from the various asymptotic forms

$$\frac{1}{k}[\delta-\delta(R)]\cong\frac{1}{k^2}\frac{8}{3}\int_R^\infty dr_1\int_0^{r_1} dr_2\,\sin^2(kr_1+\delta)e^{-2r_2}\left(\tfrac{1}{2}r_2^4+r_2^3\right)$$

$$\cong\frac{\alpha}{k^2}\int_R^\infty\frac{\sin^2(kr_1+\delta)}{r_1^4}\,dr_1, \tag{2}$$

where $\alpha=9/2$ is the polarizability of hydrogen. The right-hand side of (2) can be simply carried out to order k:

$$\frac{1}{k}[\delta-\delta(R)]=\left(\frac{\alpha}{3}\frac{a^2}{R^3}-\alpha\frac{a}{R^2}+\alpha\frac{1}{R}\right)+k\left(\frac{\pi\alpha}{3}\right)+O(k^2\ln k). \tag{3}$$

The linear term in k induces a linear term in the effective-range expansion of $k\cot\delta$:

$$k\cot\delta=-\frac{1}{a}+\left(\frac{\pi\alpha}{3a^2}\right)k+\cdots, \tag{4}$$

and the coefficient of the k term is precisely that derived by Spruch et al.[5] for the idealized problem of a particle scattered from a $-\alpha/r^4$ potential.

However, of prime significance to us is the constant term in (3), i.e., the formula

$$a=a(R)-\alpha\left(\frac{1}{3}\frac{a^2}{R^3}-\frac{a}{R^2}+\frac{1}{R}\right). \tag{5}$$

Clearly, a calculation which includes no long polarization gives rise to an approximation to $a(R)$, which will be substantially greater than the true scattering length, a.

Our own calculations were carried out to $R=20$. We obtain $a(R)=1.944$. Now applying (5) gives the final value $a=1.74$.

The same considerations also apply to the singlet scattering; however, since $a\approx 6$ is much larger in that case, the decrease in $a(R)$ will be smaller.

We expect to have accurate phase shifts for both singlet and triplet scattering shortly. These together with details of the method will be published at that time.

I should like to thank Miss Dorothy Hoover for programming the δ_0 calculation. I am greatly indebted to Dr. L. Spruch and Dr. T. F. O'Malley whose suggestion that we look at the long-range potential effect on the phase shift has led directly to this Letter. The helpful correspondence of Dr. Charles Schwartz is also acknowledged.

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PROTON SPECTRA FROM D(n, p)2n REACTION AT 14.4 Mev

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It has been pointed out[1-3] that the breakup of the deuteron by nucleons could give information on nucleon-nucleon interactions where this cannot be obtained from nucleon-nucleon scattering. Several measurements on D(n, p)2n and D(p, n)2p reactions have been performed.[4-7] At incident proton energies above 6 Mev for the D(p, n)2p reaction, the neutron energy spectra at 0° show a peak at the maximum neutron energy. This peak has been attributed[8,9] to the final-state interaction of the two protons. The present experiment was undertaken to see whether an analogous peak existed in the D(n, p)2n reaction. Such a peak could provide information on the n-n interaction.

The proton spectra have been measured using a dE/dx - E counter telescope described by Kuo, Petravić, and Turko.[10] Briefly, it consists of a CsI(Tl) scintillation counter and two proportional counters in coincidence and another proportional counter in anticoincidence. The main problem in this experiment lies in distinguishing the breakup protons from the elastically scattered deuterons. Since at the high-energy end of the spectrum, the number of deuterons exceeds that of protons by a factor of about 100, the more conventional methods of displaying E and dE/dx on an oscilloscope or multiplying electronically dE/dx and E are not reliable. The problem was overcome [as suggested by one of us (M.P.)] by displaying directly the dE/dx spectra, for fixed energy intervals, on a two-dimensional ultrasonic memory 100-channel analyzer.[11] Five dE/dx spectra corresponding to five consecutive energy intervals determined by the scintillation (E) counter could be examined simultaneously. By changing the bias at the scintillation counter output, the complete E spectrum was scanned. Figure 1 shows a typical set of spectra. The protons and deuterons could be identified without ambiguity. This was facilitated by the close agreement of the Landau theory with the experimental ΔE distributions.[10]

A flux of 1.5×10^9 neutrons/sec in 4π was obtained from the reaction T(d, n)He^4 using the 200-kev Cockcroft-Walton accelerator of Institute Ruder Bošković.[12] The neutrons were monitored by counting the associated α particles with a CsI(Tl) crystal polished to 6 mg/cm^2. The deuteron target was in the form of deuterated paraffin, 1×1 cm^2 in area and 8.4 mg/cm^2 in thickness. The experiment was performed with the counter set at 0° and 10° with respect to the incident neutrons. In the first case, the scattering angle was $4°\pm4°$ ($\pm4°$ denoting the extreme scattering angles accepted by the E counter),

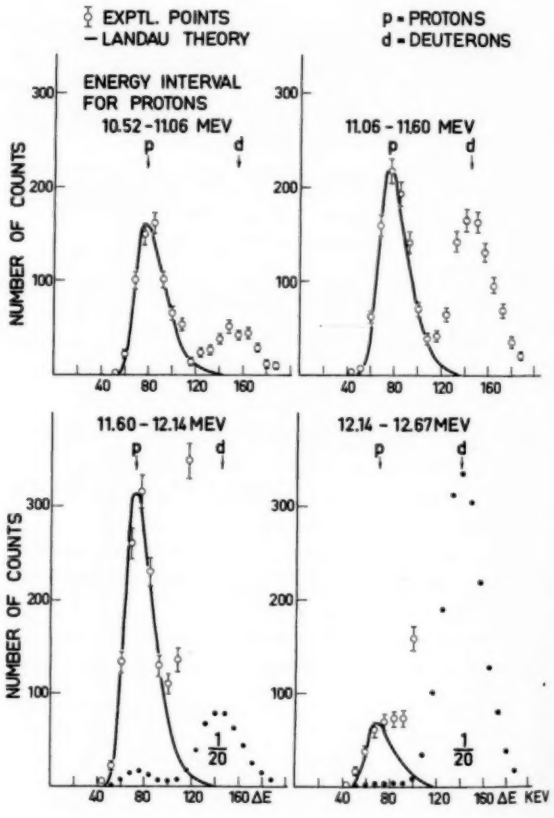

FIG. 1. Spectra of proton and deuteron energy losses (ΔE) in the dE/dx counter for fixed energy intervals in the E counter as measured by a two-dimensional ultrasonic memory 100-channel analyzer. The full curves are the predictions of the Landau-Symon theory calculated from B. B. Rossi, High-Energy Particles (Prentice-Hall Publications, Inc., New York, 1952). The errors shown are statistical errors.

and in the second case $10° \pm 8°$. Figure 2 shows the breakup proton spectra. The data have been corrected for the variation of energy interval arising from different energy losses at different energies. Neutrons degraded in energy or incident in other directions may cause a distortion in the energy spectra by two processes: firstly, by n-p scattering on the hydrogen contamination present in the deuterated paraffin target, and secondly, by producing breakup protons. The hydrogen contamination in the target has been determined to be 2.8% by measuring the elastically scattered protons and deuterons at 4°. An estimate of the degraded neutron spectrum incident in the forward direction was obtained from the n-d elastic scattering spectrum. Corrections due to these neutrons have also been applied. These corrections were largest at the low-energy end where they amounted to 5%. Relative error in the spectrum, excluding statistical error, is about 4%. The absolute cross section was determined by normalization to n-p elastic scattering using a CH_2 target. The total error in absolute cross section is 8%.

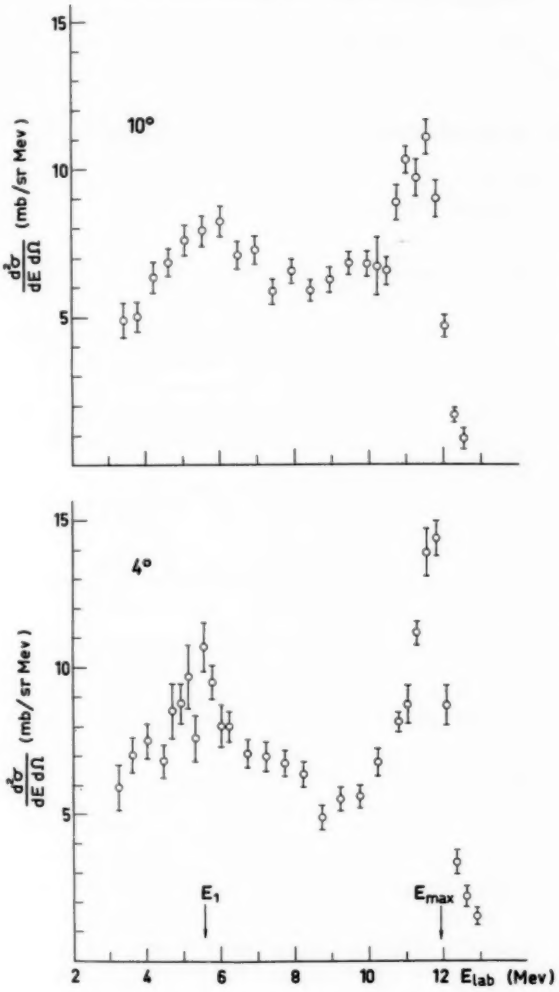

FIG. 2. Energy spectra of protons from deuteron breakup in the laboratory system. E_{max} indicates the maximum proton energy for $Q = -2.225$ Mev; this is in the case when the two neutrons are going together (unbound) backwards in the c.m. system. E_1 indicates the proton energy in the case where a neutron and the proton are going together (unbound) in the forward direction in the c.m. system. Each point represents an average over 0.54 Mev. The errors shown are statistical errors.

The proton spectrum obtained at 4° shows close similarity with the neutron spectra from the $D(p,n)2p$ reaction.[6] Although exact comparison cannot be made since the $D(p,n)2p$ data[8] are for a somewhat lower incident energy, it is possible to remark that the peak at the maximum proton energy is narrower and more pronounced in the present case. This is to be expected since Coulomb repulsion in the case of the final-state interaction of the two protons must broaden the peak.[8] Measurements at larger scattering angles which are in progress show that the peak becomes smaller as the angle increases and that at 30° (lab) it is hardly discernible. Komarov and Popova,[9] taking into account the final-state interaction between all pairs of nucleons, have obtained a very good fit to the $p+d$ data.[5,6] They point out that a similar calculation can be made for $n+d$ breakup where the low-energy part of the proton spectrum could be attributed to n-p interaction and the high-energy peak to n-n interaction. In fitting the high-energy peak, the n-n scattering length could be deduced.

Without a detailed theoretical analysis of the present data, one cannot draw a conclusion about the existence of a bound state of the di-neutron.

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RADII OF OPTICAL-MODEL POTENTIALS IN PROTON SCATTERING AT 183 Mev

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(Received February 21, 1961)

Previous analyses[1,2] of the elastic scattering of 150-300 Mev protons by nuclei using the optical model of the interaction have shown that it is able to give a good account of the existing experimental differential cross sections and polarizations. In most cases, however, the experimental data do not extend much beyond the first diffraction minimum, and so do not accurately determine the parameters of the optical-model potential.

Recently, Johansson and Svanberg[4] have made a series of accurate measurements of 183-Mev protons elastically scattered by a range of eight nuclei from lithium to gold. Particular care was taken to eliminate inelastically scattered protons, and the differential cross sections were obtained out to around 70°. Measurements of the reaction cross sections were also made and these, combined with the existing polarization data, make a more complete set of data for optical-model analysis than has hitherto been available. We have found that the elastic scattering and polarization can be simultaneously well described only if the imaginary part of the optical-model potential is given a significantly greater radius than the real part.

The optical-model potential was taken to have the usual form,

$$V(r) = V_{\mathrm{Coul}}(r) + Uf_1(r) + iW[\alpha f_2(r) + (1-\alpha)g(r)] + \left(\frac{\hbar}{m_\pi c}\right)^2 \left[U_S \frac{df_3(r)}{dr} + iW_S \frac{df_4(r)}{dr}\right] \frac{1}{r}\vec{l}\cdot\vec{\sigma},$$

where $f(r)$ and $g(r)$ are Saxon-Woods and Gaussian form factors, and the other symbols have their usual meanings.[4] All calculations were done on the Oxford University Mercury computer using methods described elsewhere.[5]

Preliminary attempts to fit the data were concentrated on aluminum, partly because it is a fairly light nucleus and partly because the difference between the experimental data and previous optical-model calculations at[2] 160 Mev is greatest for this nucleus.

The most striking feature of the experimental differential cross section [Fig. 1(a)] is the virtual absence of oscillatory structure, in sharp contrast to the considerable oscillations shown by the optical-model cross section with the potentials used previously. This effect is most marked beyond the first minimum and so has not been noticed previously. The absence of oscillations cannot be attributed to experimental smearing, as particular care was taken to achieve good angular (± 0.4°) and energy (± 0.5-Mev) resolution.

The first aim of the present work was to resolve this discrepancy. To facilitate comparison with previous work, as many parameters as possible were kept the same as before. The starting potentials were taken to be $U = -18$, $W = -12$, $U_S = 2.5$, $W_S = -1$ Mev, and all radii were fixed at $R = 1.25\,A^{1/3}$ f, with diffuseness parameters $a = 0.65$ f and $b = 1.0$ f for the Saxon-Woods and Gaussian form factors. With $\alpha = 1$, corresponding to a Saxon-Woods absorption potential, the parameters U, W, a_1, a_2, and $a_3 = a_4$ were varied to see if the oscillations could be removed, but without success. Further calculations were made with the Gaussian form for the absorbing part of the central potential, and with an equal mixture of Gaussian and Saxon-Woods forms ($\alpha = 0.5$). In each case the differ-

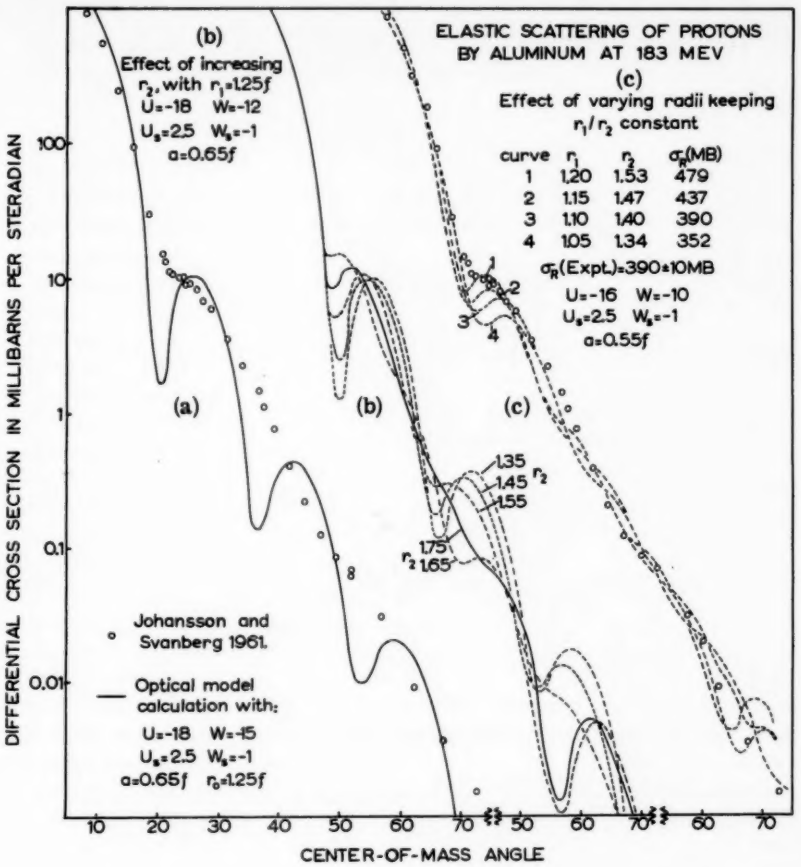

FIG. 1. (a) Experimental differential cross sections for the elastic scattering of 183-Mev protons by aluminum compared with an optical-model calculation with equal radii for the real and imaginary parts of the potential. (b) Calculated differential cross sections for the same interaction with increasing values for the radius of the absorbing potential. (c) The same for a series of radii, with $r_2 = 1.27\, r_1$.

ential cross sections showed substantial oscillations, in disagreement with the experimental data.

The effect of different radii for the real and imaginary parts of the potential was then studied. Keeping the radius of the real part at $r_1 = 1.25$ f, the radius of the absorbing part r_2 was varied from 1.25 to 1.75 f for the Saxon-Woods form factor ($\alpha = 1$). Figure 1(b) shows that when $r_2 = 1.75$ f, the differential cross section falls smoothly with angle from 25° to 55°, like the experimental data. This value of the radius, however, gives too large a value for the reaction cross section. The radii were, therefore, progressively reduced in the same ratio until the reaction cross section attained approximately the correct value; this happens when $r_1 = 1.10$ f, and $r_2 = 1.40$ f. The results of these calculations are given in Fig. 1(c), and show that the smooth fall of the differential cross section is retained. The slight oscillation around 20-30° is qualitatively given by the model, and it is convenient to adjust the radii until the maximum of this oscillation has the experimental value. The reaction cross section can be brought back to the correct value by adjusting W, the strength of the absorbing potential. Similar calculations were made for $\alpha = 0$ and $\alpha = 0.5$, but in all cases the differential cross section showed oscillations greater than those observed experimentally. In all this work the radii of the spin-orbit potentials r_3 and r_4 were kept equal to the radii of the corresponding central potentials r_1 and r_2, respectively.

If the polarization given by the best of these potentials is compared with experiment (Fig. 2), it is found to have the required minimum around 23° that was not given by any potentials with $r_1 = r_2$. The calculated minimum is rather deeper than the experimental, but it is possible that adjustment of the spin-orbit potentials could remove the remaining discrepancy. Thus in fitting

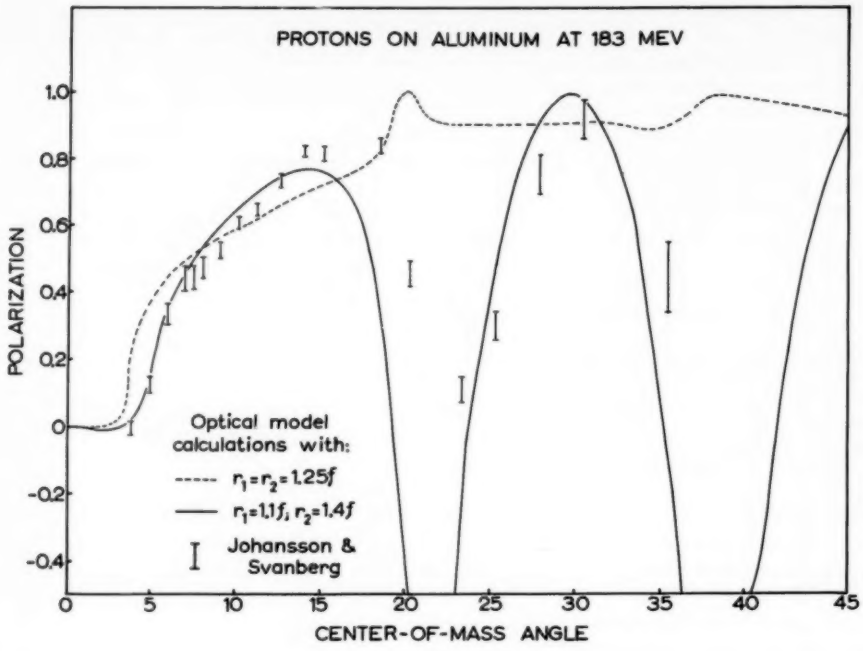

FIG. 2. Experimental polarization for the elastic scattering of 183-Mev protons by aluminum, compared with the optical-model calculations using the same potentials as in Figs. 1(a) and 1(c) (Curve 3).

the differential and reaction cross sections the most serious polarization discrepancy has been removed as well.

These potentials were then used to calculate the cross sections and polarizations for the remaining nuclei Li, Be, C, Ca, Fe, In, and Au, and in each case good qualitative agreement with the experimental data was found. In particular, the smooth fall of the differential cross section is well reproduced. For each of these interactions the fit to the experimental data can be improved by systematic fine adjustment of the parameters. This work is still in progress and will be reported on completion.[3]

Although the parameter space has not been completely explored, the present preliminary work thus suggests that in energetic proton interactions with medium-weight nuclei the absorbing part of the local potential extends some 30% beyond the real part, and that it is similar to the Saxon-Woods form.

I am grateful to Dr. A. Johansson for sending me his experimental results before publication, to Dr. B. Buck and Mr. R. N. Maddison for their work on the computer programme, to the Director of the Oxford University Computing Laboratory for the use of the Mercury computer, and to the technical staff for their assistance.

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NUCLEAR ORIENTATION OF PARAMAGNETIC IMPURITY IONS*

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(Received February 21, 1961)

The well-known Gorter-Rose method of nuclear orientation has been used with considerable success in recent years.[1-4] The method as used consists in cooling, by adiabatic demagnetization, a paramagnetic salt into which ions containing radioactive nuclei have been incorporated in lattice positions. These ions are polarized by a small applied magnetic field, and they in turn polarize their nuclei. A limitation of the method is that only nuclei of elements which form paramagnetic ions isomorphous with those of certain "good" cooling salts can be studied.

A substantial extension of this technique would result if nuclei of all elements that form paramagnetic ions could be oriented in this way. The purpose of this Letter is to point out that this extension is made possible by incorporating the ions of interest as impurities into single crystals of cerium magnesium nitrate (CMN). The impurity ions may be interstitial or they may be trapped in tiny "brine holes." At any rate, they should be in thermal equilibrium with the lattice after demagnetization and can be polarized in fields of several hundred oersteds at temperatures below 0.01°K. To test this hypothesis, we have carried out nuclear-orientation experiments with Cr^{51} in CMN and have detected nuclear orientation by measuring the anisotropy of the 325-kev gamma rays.

The specimen consisted of a 5-g single crystal of CMN grown from a solution containing small amounts of Cr^{51} along with about 1 mg of Cr^{+++} carrier. It was mounted in a demagnetization cryostat described elsewhere,[5] and cooled by adiabatic demagnetization to ~0.003°K. A polarizing field was applied along the trigonal axis of the crystal, and the gamma-ray counting rates were measured along and perpendicular to the field direction. The magnetic temperature of the crystal was monitored continuously during the counting period. The results for two polarizing fields are shown in Fig. 1. The experiments were repeated with a second, independently grown crystal, and the data were found to be reproducible. Nuclear-orientation experiments in zero applied magnetic field show that the gamma-ray emission from Cr^{51} is still anisotropic, the normalized intensities being about 1.03 along the axis and 0.98 perpendicular to the axis, at the lowest temperatures reached.

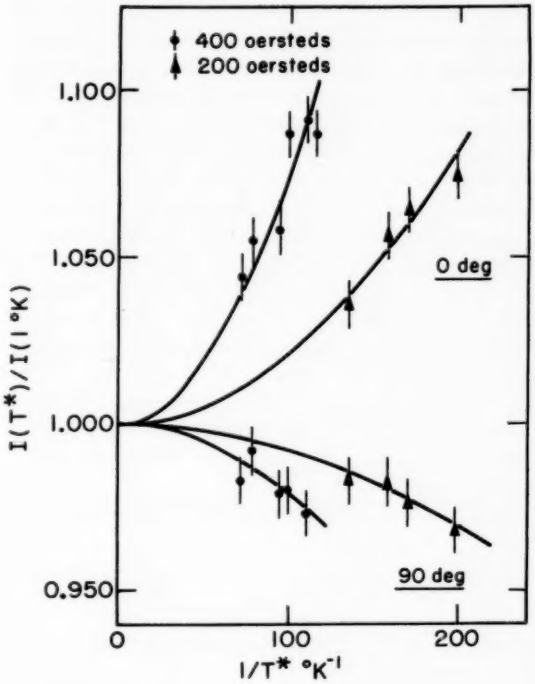

FIG. 1. Experimental results for nuclear orientation of Cr^{51} in cerium magnesium nitrate as a function of the reciprocal magnetic temperature, $1/T^*$. The ordinate is the ratio of the intensity of 325-kev gamma rays at temperature T^* to the intensity at 1°K. Data shown are from measurements at 0 and 90 deg to the crystal trigonal axis, for two values of the magnetic field applied along the axis. The uncertainties indicated are statistical standard deviations. The curves drawn through the points are proportional to $(1/T^*)^2$.

Although the main object of this experiment was to test the above-mentioned nuclear-orientation mechanism, some nuclear information was obtained from a preliminary analysis of the results. Using our data for a polarizing field of 400 oersteds, where nondiagonal interactions should be least important, together with the measured partial $E2$ and total lifetimes[6,7] of

the 325-kev state in V^{51}, we find: (1) The $E2$-$M1$ mixing ratio of the 325-kev gamma ray of V^{51} has the value $\delta(E2/M1) = +0.37 \pm 0.04$; (2) a spin assignment of 7/2- is absolutely ruled out for this state; and (3) a lower limit of $|\mu| \geq 0.8$ nm can be set for the magnetic moment of Cr^{51}.

The evidence which shows that Cr^{+++} does not go into lattice sites in CMN is derived from experiments in which we attempted to grow several percent chromium into cerium magnesium nitrate and lanthanum magnesium nitrate. In these experiments, the double nitrate crystals were grown from solutions containing relatively high concentrations of Cr^{+++}. The crystals obtained were analyzed spectroscopically for chromium with the result that portions of crystals that were cloudy and slightly blue contained small amounts ($\sim 0.1\%$) of chromium. However, those portions of crystals which were optically clear and colorless contained no detectable traces of chromium. We conclude that Cr^{+++} does not substitute in lattice positions, but is probably incorporated into the crystal in "brine holes" as growth proceeds.

Considerable development may be necessary before this technique can be used to obtain reliable values of nuclear moments. It should be immediately useful, however, for investigating other nuclear properties. In particular, nuclei of elements in the $4d$, $5d$, and $5f$ transition series should be within the scope of the method.

We thank George Shalimoff for performing the analyses of the double nitrate crystals.

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*Work supported by the U. S. Atomic Energy Commission.

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SATELLITE DETERMINATION OF HEAVY PRIMARY COSMIC-RAY SPECTRUM*

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(Received March 13, 1961)

This note constitutes a preliminary report of some results of an experiment conducted aboard the satellite Explorer VII (1959 Iota) to investigate possible time variations in the flux of heavy nuclei ($Z \geq 6$) in the primary cosmic radiation. In particular, a detailed analysis of data recorded during two different periods in the lifetime of the experiment has provided values of the exponent, γ, in the integral magnetic rigidity spectrum expressed in the form:

$$N(>pc/Ze) = k(pc/Ze)^{-\gamma}. \qquad (1)$$

The energy spectra of the multiply-charged primary cosmic rays have previously been determined either by means of the latitude effect, or on the basis of observations at a single location.[1] The former method involves the comparison of flux measurements obtained at different times with nuclear emulsions or counter arrangements flown at several places. Uncertainties in the geomagnetic cutoff energies,[2] the possible effects of time variations, and various other considerations add to the scatter in the results arising solely from statistical fluctuations. Determinations based upon energy measurements on individual nuclei observed during a single balloon flight, while consistent with those utilizing the latitude effect, also display a considerable spread, and neither approach is particularly amenable to providing information concerning possible time variations in the spectrum of the

M and H nuclei.

A previous investigation of heavy nuclei by means of equipment carried aboard a satellite was concerned exclusively with relative abundances. The ratio of the fluxes of nuclei with $Z > 15$ and $Z > 30$, as registered by a Čerenkov counter aboard Sputnik III, has been reported by Kurnosova, Razorenov, and Fradkin.[3]

The detector in the present experiment comprised a pulse ionization chamber fabricated of magnesium. The experimental arrangement is described in detail elsewhere.[4] The data were recorded without any long-term on-board storage via the facilities of a world-wide network of receiving stations established and operated by NASA. This coverage made it possible continuously to measure the flux of particles with $Z \geq 6$ over a considerable portion of the satellite's orbit (inclination 50.32°).

In contrast with apparatus designed to measure the primary proton flux in the vicinity of the earth, this system is characterized by a high rejection ratio for the particles in the radiation belts. As a consequence, despite the fact that over much of its orbit the satellite was in a region in which the flux of trapped radiation is quite large, most of the measurements obtained with this equipment were unaffected by this background. Thus, observations of the flux of heavy primaries over a considerable portion of the earth at altitudes ranging from 530 km to 1100 km have been obtained.

The present data permit the comparison on a global scale of alternative approximations which have been invoked for representing the effects of the geomagnetic field upon cosmic-ray cutoff rigidities.[2] In Figs. 1 and 2, the relative intensity of heavy nuclei during the period October 13-24,

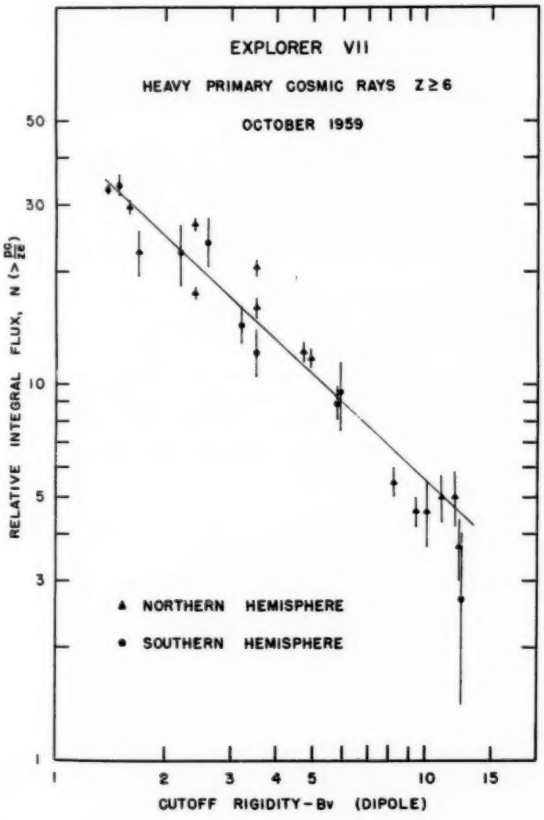

FIG. 1. Relative intensity of heavy nuclei during the period October 13-24, 1959, plotted as a function of cutoff rigidity (dipole approximation).

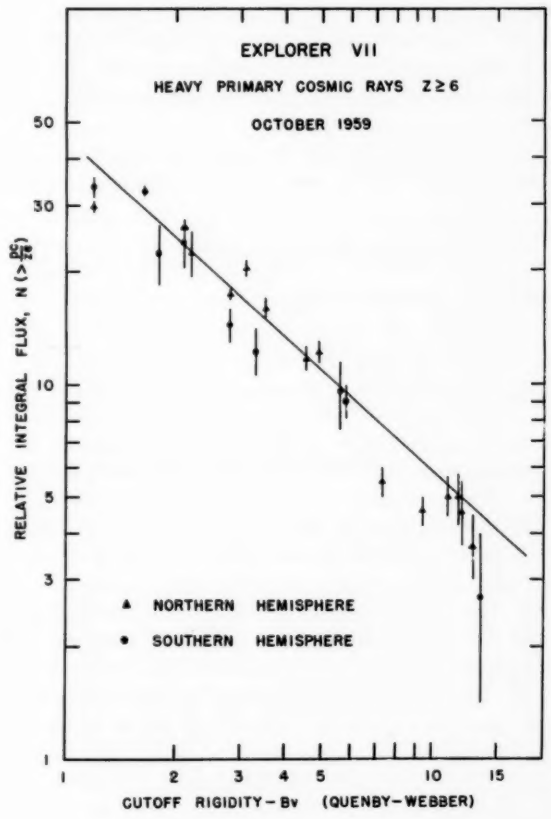

FIG. 2. Relative intensity of heavy nuclei during the period October 13-24, 1959, as a function of cutoff rigidity (Quenby-Webber formulation).

1959, has been plotted as a function of cutoff rigidity calculated on the basis of the conventional centered dipole field approximation,[5] and the dipole plus nondipole formulation of Quenby and Webber,[6] respectively.

The corresponding values of γ [Eq. (1)] were determined by fitting a straight line to the data by the method of least squares. The omnidirectional response of the detector and the variation of the geomagnetic field as a function of altitude have been taken into account in both cases in the computation of the cutoff rigidity applicable to each latitude-longitude-altitude box. Data recorded during the period April 13-20, 1960, were analyzed in a similar manner. The results are summarized in Table I. The indicated uncertainties were derived from the least-squares analyses solely on the basis of the deviations in the ordinates. For the present purposes, the two approximations appear equally acceptable.

Figures 1 and 2, and Table I, reveal that, although individual cutoffs assigned to particular orbital positions may be strongly dependent upon the field approximation at some locations, the net effect upon the value of γ based upon a global sampling is not significant.

Furthermore, within the experimental limitations, the value of γ was the same during the two designated periods in October, 1959, and April, 1960. The average intensity of the nucleonic component at sea level as measured by neutron monitors was about 1% lower during the latter interval.

Table I. Determinations of γ for $M+H$ nuclei.

Field approximation	γ(October, 1959)	γ(April, 1960)
Dipole	0.91 ± 0.07	0.80 ± 0.07
Dipole + nondipole	0.89 ± 0.07	0.90 ± 0.06

Earlier determinations of the spectrum of heavy nuclei have usually been expressed in the form $J(>T) = cT^{-m}$, where T is total energy (kinetic + rest) in Bev/nucleon. On the basis of the present data, the value of the exponent, m, in this representation is 1.2. Unfortunately, comparison of this result, which pertains to a period near an outstanding solar maximum, with previous measurements characteristic of a low level of solar activity is complicated by various factors. Nevertheless, it is interesting to note that it falls within the range of values summarized by Waddington.[1]

It is a pleasure to thank Kazuo Nagashima for helpful discussions. Appreciation is expressed to Herman LaGow of NASA, and to all who helped in carrying out the Explorer VII program.

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*Assisted by the National Science Foundation.

†Deceased.

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EXPERIMENTAL RESULTS ON THE π-π CROSS SECTION*

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(Received March 2, 1961)

At the 1960 Rochester Conference,[1] we reported our first experimental results on the π-π cross section, which we obtained by applying the Chew-Low extrapolation method[2] to about 700 π-p inelastic scatterings. These events were analyzed in photographs taken with the Alvarez 72-inch hydrogen bubble chamber, which was exposed to a 1.03-Bev/c π^- beam at the Bevatron. We now have a total of 1275 inelastic events with the proton stopping in the chamber. In this paper we will use the notation, the selection and evaluation criteria, and the extrapolation procedures which have been discussed in reference 1.

Figure 1 shows our experimental distributions,

$$F(p^2, \omega^2) = (p^2+1)^2 \partial^2\sigma(p^2, \omega^2)/\partial p^2 \partial \omega^2,$$

as functions of the four-momentum transfer p^2

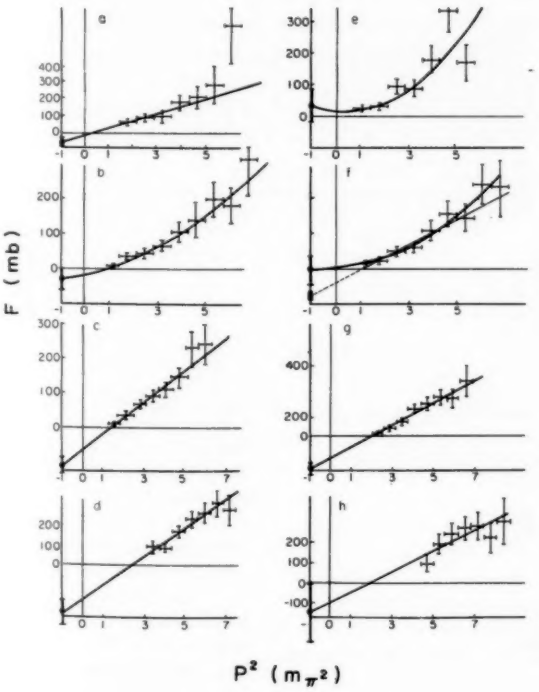

FIG. 1. Extrapolation curves $F(p^2, \omega^2)$ at fixed ω^2. (a) $\omega^2 = 5$ to 8.2 m_π^2; (b) $\omega^2 = 11$ to 13.7; (c) $\omega^2 = 16.5$ to 19.2; (d) $\omega^2 = 22$ to 24.7; (e) $\omega^2 = 8.2$ to 11; (f) $\omega^2 = 13.7$ to 16.5; (g) $\omega^2 = 19.2$ to 22; (h) $\omega^2 = 24.7$ to 27.5.

for eight different intervals of ω^2, the invariant mass of the di-pion. We express p^2 and ω^2 in units of the pion mass squared. Fitting polynomials in (p^2+1) of several orders through these data, we obtain results as follows.

1. Whereas at the time of the Rochester Conference a first-order polynomial adequately fitted the data for all the intervals of ω^2, we find that a χ^2 test now reveals the necessity of a quadratic fit for the low-ω^2 intervals. In the higher ω^2 intervals, although we have more events than at the lower ω^2 intervals, the linear fits are still exceedingly good. We therefore take the linear values for the extrapolation π-π cross section at the higher energy.

2. Also at the time of the Rochester Conference some of the fitted curves had the unphysical feature that they went negative in the physical region. This was possible because we do not have events right up to the edge of the physical region. Since we now require quadratic fits for the lower ω^2 intervals, this difficulty has been removed in all cases except for the lowest ω^2 interval (5.5 to 7.8 m_π^2). For this interval only we had to apply a constraint forcing the fitted curve to go through the end of the physical region. We then find that a linear curve with this added constraint gives a good fit. In all other intervals the fitted curve remains positive in the physical region without this constraint.

In order to get some information about the π-π interaction from a very small number of events, Bonsignori and Selleri[3-5] have made the assumption that formula (3.13) of reference 2 (which is exactly correct only at the pole $p^2 = -1$) approximately describes the p^2 dependence at the beginning of the physical region. With this assumption an average π-π cross section has been determined. The assumption means that all other contributions to the amplitude for single-pion production (p^2 dependence of the vertex functions of the single-pion exchange diagram, exchanges of 3, 5, or more pions, and production by collision of the pion with the nucleon core rather than with the cloud) have been neglected in comparison with the single-pion exchange at the pole. With our present statistics we are able to check the assumption of Bonsignori and Selleri. If it were justified, it should be possi-

ble [as one can see from (3.12) in reference 2] to fit the experimental distributions (or the first part of them) of Fig. 1 by a straight line passing through zero for $p^2=0$. Our data show that this is not possible. Thus, nonpole terms are important and an extrapolation requiring a large number of events is necessary to extract the pole term.

Frazer and Fulco[6] could explain the vector part of the electromagnetic structure of the nucleon assuming a strong π-π resonance in the $T=J=1$ state at $\omega^2=11$. The π^--π^0 amplitude is 50% isotopic spin 1 and 50% isotopic spin 2. We find no evidence for a π-π resonance of the width and location predicted in reference 6.

Our data do show a rise in the π-π cross section starting at $\omega^2=17$, reaching a value of the order of 200 mb at $\omega^2=20$ to 22. (See Fig. 2.)

However, one must remember that it is just in this region of ω^2 that our extrapolation distance begins to get larger, making the extrapolation procedure less conclusive. We are now scanning film obtained at 1.275 Bev/c incident momentum in order to reduce the extrapolation distance in this region. Also, for ω^2 greater than 9, we have a background of events coming from the reaction $\pi^-+p\rightarrow\pi^-+p+2\pi^0$. These events do not have a pole at $p^2=-1$, but come from a branch cut. We are eliminating this background by means of a kinematical fit.

On the other hand, Bowcock et al.[7] found on a later analysis of the nucleon electromagnetic structure and the low-energy pion-nucleon phase shifts that the Frazer-Fulco resonance should be shifted to about $\omega_{res}^2=22$. This is consistent with our present results. If we assume that our data peak at $\omega^2=20$ to 22 (our incident energy is insufficient to examine the high-energy side of the peak), then the height is in accord with $(2J+1)4\pi\lambda^2\sin^2\delta_1$ for a p-state resonance—that is, $\sin^2\delta_1=1$. Our half-width (obtained from the low-energy side) is approximately $5m_\pi{}^2$. Of course, our data do not rule out a nonresonant rise in the cross section composed of s, p, d, f, $\cdots$ states which just happens to satisfy $12\pi\lambda^2$ at $\omega^2=20$ to 22. We are currently evaluating the differential cross section in the region $\omega^2=20$. We hope to resolve this experimental ambiguity in the very near future.

We would like to thank Professor Luis W. Alvarez for his great interest and encouragement throughout the experiment. It is a pleasure to thank Professor Frank S. Crawford, Jr., who designed the beam, as well as Professor Arthur H. Rosenfeld and other members of the Alvarez group, for many interesting and stimulating discussions. We are indebted to Professor Goeffrey F. Chew and to Dr. James S. Ball for several discussions on the theoretical aspects and to Dr. Herbert M. Steiner for useful comments. Finally, we want to thank our scanners for their help in finding and analyzing the events.

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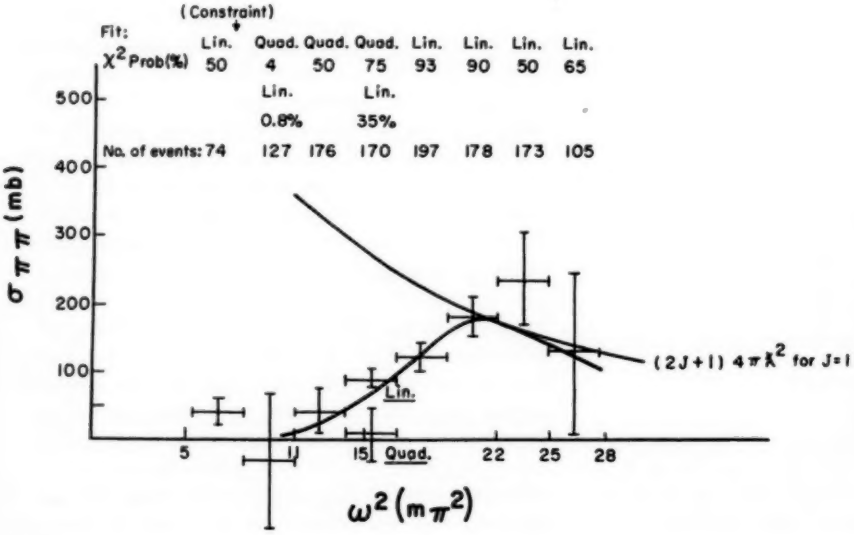

FIG. 2. The π^--π^0 cross section as a function of the total di-pion mass squared as determined by the Chew-Low method. Also shown are the maximum height of a p-state resonance and the shape of the Frazer-Fulco resonance [W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 367 (1959), Eq. (10)], assuming the parameters $v_r=3.5$, $\Gamma=0.3$.

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ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON AND PION-PION INTERACTION

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We wish to propose a simple model for the electromagnetic structure of the nucleon, based on dispersion theory and on a strong pion-pion interaction. The model is a synthesis of several theoretical ideas proposed by Frazer and Fulco,[6] Nambu,[2] and Chew.[3]

Let us first of all summarize some general properties of the nucleon form factors. We write the interaction of the nucleon with the electromagnetic field in the form:

$$\langle p'|j_\mu|p\rangle A_\mu = i\bar{u}(p')[G_1(t)\gamma_\mu + G_2(t)\sigma_{\mu\nu}k_\nu]u(p)A_\mu, \quad (1)$$

where p', p, and k are the four-momenta of the final nucleon, initial nucleon, and photon, respectively, and $t=k^2=(p'-p)^2$. The G_i still are operators in the isospin space:

$$G_i = G_i^S + G_i^V \tau_3,$$

and so

$$G_i^p = G_i^S + G_i^V; \quad G_i^n = G_i^S - G_i^V.$$

As is well known, the separation into the isoscalar and the isovector current is very useful because only an even number of pions contribute to G^V and an odd number to G^S. At $t=0$ the G_i functions tend to the static charge and magnetic moment of the nucleon:

$$G_1^p(0)=e, \quad G_1^n(0)=0,$$

$$G_2^p(0)=\mu_p=eg_p/2M, \quad G_2^n(0)=\mu_n=eg_n/2M,$$

$$G_1^S(0)=G_1^V(0)=e/2,$$

$$G_2^S(0)=(\mu_p+\mu_n)/2=eg_S/2M,$$

$$G_2^V(0)=(\mu_p-\mu_n)/2=eg_V/2M,$$

$$g_p=1.79, \quad g_n=-1.91,$$

$$g_S=-0.06, \quad g_V=1.85, \quad (2)$$

The functions $G(t)$ are related to the usual Hofstadter form factors $F(t)$ by the following definitions:

$$G_i^{p,n}(t)=G_i^{p,n}(0)F_i^{p,n}(t). \quad (3)$$

Dispersion theory allows one to write the different functions $G(t)$ in the following form[4]:

$$G(t)=\frac{1}{\pi}\int_0^\infty \frac{g(t')}{t'-t}dt'. \quad (4)$$

The spectral functions $g(t)$ are of fundamental theoretical importance because they are related to the weight with which the different many-particle states contribute to the nucleon form factors. Therefore, if there is no bound state formed by pions, $g(t)$ will be different from zero only for t larger than $4m_\pi^2$ for the vector part and $9m_\pi^2$ for the scalar part.

If there is no strong correlation between the pions, $g(t)$ is related just to the statistical weight of the many-particle state with c.m. energy $E=\sqrt{t}$ and therefore will be a smoothly increasing function of t starting from zero.

If on the other hand, there is a strong correlation between the pions, due to a resonance with an energy E_R, the spectral function will exhibit a maximum for $t=t_R=E_R^2$. This result was first shown by Frazer and Fulco[1] and is schematically illustrated in Fig. 1.

Therefore, in a model without a strong pion-pion correlation the spectral function $g(t)$ will be dominated by the large values of t and $G(t)$ will have little dependence on t. The discrepancy of this model from the experimental data is discussed by Drell.[5]

On the other hand, a strong-correlation model leads to a rapid variation of $G(t)$ as suggested by experiment. Since for observable values of t ($t \leq 0$) the dispersion denominator in Eq. (4) is always positive, if the resonance state has a reasonable width ($< \frac{1}{2}m_\pi$), its effect can be well approximated by means of $\alpha/(t_R - t)$, where t_R is the resonance position and α is the area under the resonance curve.

Let us now discuss briefly the experimental results for the nucleon form factors at low momentum transfers. For $-t<10m_\pi^2$ the nucleon form factors were roughly given as follows:

$$F_1^p(t) \simeq F_2^p(t) \simeq F_2^n(t) \equiv F(t),$$

$$F_1^n(t) \simeq 0. \tag{5}$$

Many different analytic functions were proposed for $F(t)$ but a particularly good fit was obtained either with an exponential form or with the form proposed by Clementel and Villi[6]:

$$F(t) = 0.2 + \frac{1.2}{1-(t/22m_\pi^2)}. \tag{6}$$

From our theoretical point of view, it is very difficult to understand an exponential form factor; on the other hand, the Clementel-Villi model can be naturally understood on the basis

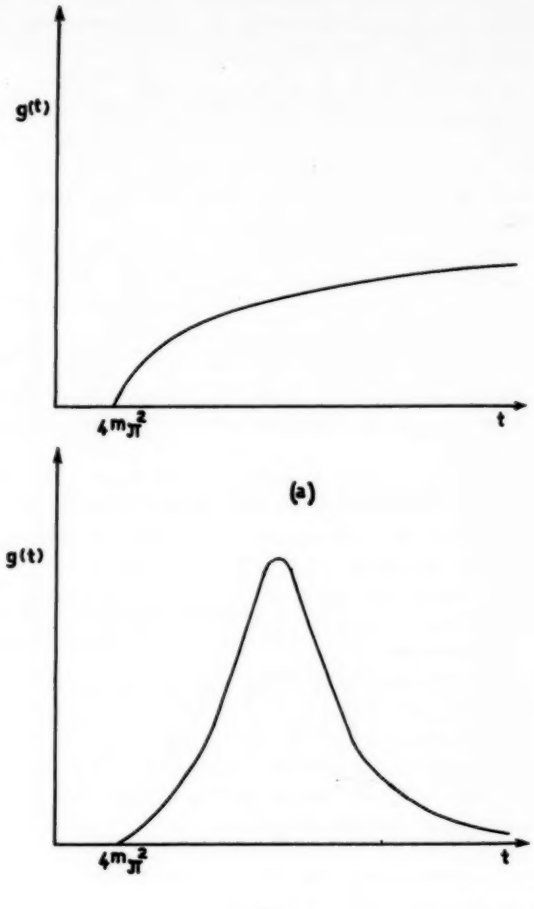

FIG. 1. Schematic representations of $g(t)$ in arbitrary scale. (a) Uncorrelated pions; (b) strong pion-pion resonance.

of a resonant state of energy $4.7m_\pi$. The constant term appearing in Eq. (6) represents the total contribution of the higher t states which in the low-momentum-transfer region is approximately constant. Thus Eqs. (5) and (6) indicate that it is possible to interpret both isovector form factors F_1^V and F_2^V by means of the approximate form, which has a pole at $t_R \simeq 22m_\pi^2$:

$$G_1^V \simeq \frac{e}{2}\left(-0.2 + \frac{1.2}{1-(t/22m_\pi^2)}\right),$$

$$G_2^V \simeq \frac{eg_V}{2M}\left(-0.2 + \frac{1.2}{1-(t/22m_\pi^2)}\right). \tag{7}$$

By taking this attitude, the resonant state at $E_R \simeq 4.7 m_\pi$ will be attributed to a $T=1$, $J=1$ two-pion state.

Such a resonant state, at this energy, has been observed in different experiments[7] on pion production by pions, and its parameters, deduced from Eq. (7), have been used[8] to explain satisfactorily the low-energy behavior of the pion-nucleon scattering phase shifts.

The isoscalar charge form factor $G_1^S(t)$ must be of the same order of magnitude of $G_1^V(t)$, in order to give a vanishing neutron charge distribution. This means that we have to expect important low-t contributions to the isoscalar form factors. The isoscalar magnetic form factor is experimentally very small and little is known about its energy variation.

The recent[9] experimental data on electron-proton scattering at larger values of the momentum transfer stress the need of having an important low-mass contribution to the isoscalar charge form factor. It is found that the charge form factor of the proton (Fig. 2) deviates strongly from Eq. (7) and it would be impossible to fit it with any reasonable form with only one pole.

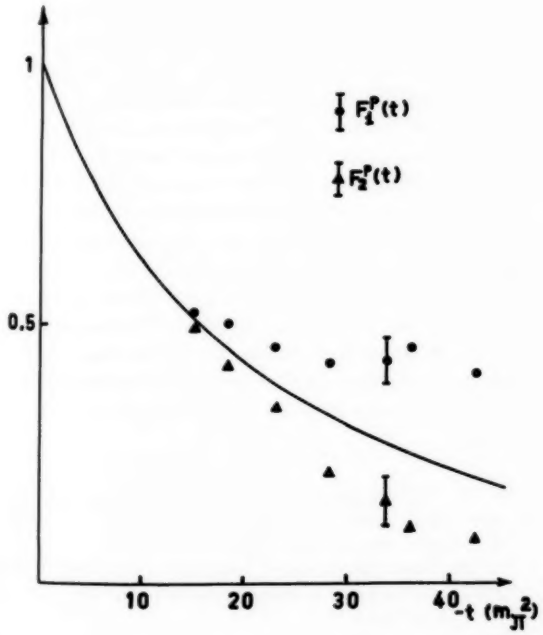

FIG. 2. Proton form factors at high momentum transfer (from reference 9). The solid line represents the Clementel and Villi model [in Eq. (6)].

A reasonable extension of the preceding ideas is to assume that G_1^V is still approximately given by Eq. (7) with the pole at $t_V \simeq 22 m_\pi^2$, which is the value suggested by other experimental information. In this manner the flattening of G_1^p will be essentially due to the effect of G_1^S. Thus G_1^S must be composed of an almost constant part and a part which goes to zero much faster than G_1^V. This means that the charge form factor of the neutron will be positive at large values of t. This result seems to be confirmed by recent experiments on the neutron.[10]

If we tentatively attribute an average mass to the decreasing part of G_1^S, we have something like $8 m_\pi^2$, certainly less than $22 m_\pi^2$. This shows that it would be very difficult to explain the rapid variation of G_1^S on the basis of a simple statistical formula for $g_1^S(t)$, since the spectral integral will only start at $9 m_\pi^2$. We thus expect the existence of a $T=0$, $J=1$ three-pion resonance (or bound state if $t_S < 9 m_\pi^2$). The magnetic isoscalar part must also have a pole at the same position of t_S, but the present experimental information about it is certainly not enough to allow detection of its effects.

From the preceding discussion we expect expressions for G_1^S which have similar form to those for G_1^V. This leads to the following general form for the nucleon form factors:

$$G_1^V = \frac{e}{2}\left[(1-a_V) + \frac{a_V}{1-(t/t_V)}\right],$$

$$G_1^S = \frac{e}{2}\left[(1-a_S) + \frac{a_S}{1-(t/t_S)}\right],$$

$$G_2^V = \frac{e g_V}{2M}\left[(1-b_V) + \frac{b_V}{1-(t/t_V)}\right],$$

$$G_2^S = \frac{e g_S}{2M}\left[(1-b_S) + \frac{b_S}{1-(t/t_S)}\right], \qquad (8)$$

where t_V and t_S represent the position of the isovector and isoscalar unstable particles, respectively.

The residues of the poles a_V, b_V, a_S, b_S are connected with the constants appearing in similar terms giving the effect of pion-pion interaction in π-N scattering, N-N scattering, and photoproduction.[7,11] The validity of the present model can thus be checked by trying a general fit of many different sets of experimental data using the same phenomenological parameters.

One of the best known experimental properties of the form factors is that the mean charge radius of the neutron is zero. This leads to

$$a_S/t_S \simeq a_V/t_V = a. \tag{9}$$

As a consequence we have

$$G_1^p = e\left[1 + \frac{a}{2}\left(\frac{t_S}{t_S - t} + \frac{t_V}{t_V - t}\right)t\right];$$

$$G_1^n = -e\,\frac{a}{2}\left(\frac{t_V}{t_V - t} - \frac{t_S}{t_S - t}\right)t. \tag{10}$$

This means that we have in our model five independent parameters (a, t_V, t_S, b_S, and b_V), for one of which (t_V) we know the approximate value from independent experiments.

Some preliminary determinations[10,12] of the parameters contained in Eq. (8), using information from the proton and neutron form factors, confirm the validity of the present model. We wish to stress a very important consequence of the fact that t_S turns out to be smaller than t_V. From Eq. (10) one sees that the outer part of the charge distribution of the neutron is positive in contrast with what one would obtain on the basis of a model without strong pion-pion interaction.[13]

It is not surprising that, in a strong $T=1$ pion-pion interaction model, the three-pion state has a mass lower than the two-pion state. The existence of a resonance in a $T=1$, $J=1$ two-pion state forces the three-pion state to be in a $T=0$, $J=1$ state in which all the three pions are two-by-two resonating in a $T=1$, $J=1$ state.[3] Moreover the $T=0$, $J=1$ three-pion state is a completely saturated unit because if we add an extra pion it will be in a $T=0$ or 2 state with respect to the others. So, if we believe that there is a strong attraction in the $T=1$ state only, we can expect that the four-, five-, etc., pion states are not strongly correlated at low energy.

The possibility of detecting experimentally the $T=0$ unstable particle (which we shall denote by ρ) is discussed in references 2 and 3. A very interesting possibility is to identify it with the $T=0$ particle observed by Abashian et al.,[14] who find $t_S \simeq 5$. The choice $J=1$ for ρ was suggested by Chew[15] in order to explain the small effect of this particle in the decay spectrum of the K meson.

The spin of ρ together with the small available phase space would make the decay probability $K \to \rho + \pi$ very small but not impossible to detect.

On the other hand, preliminary[10,12] estimates show that a value $t_S \simeq 5$ is not incompatible with the experimental data on the form factors. There also seems to be some indication in favor of $t_S \simeq 5$ coming from the theoretical analysis of the electromagnetic form factor of the α particle.[16]

Another important indication coming from the qualitative determination of the parameters of Eq. (8) is that large constant terms are needed; in other words, the two- and three-pion resonating states are not sufficient to describe the nucleon completely.

One could take two attitudes: The first is that the dispersion relations make sense only in the subtracted version and the necessary constant terms are not linked to any observable effect; the second point of view is that the form factors tend to zero for $t \to \infty$ and the constants represent contributions of high-energy states. Such contributions, which are approximately constant at low momentum transfer, will show their t dependence as the momentum transfer increases. Being in favor of the second point of view (at least so far as the magnetic form factor is concerned), we are convinced that our knowledge of the nucleon structure is still incomplete and that higher energy electron-nucleon scattering will be of great importance in clarifying the many-particle contributions to the nucleon structure.

We wish to thank Professor R. Hofstadter for having communicated to us his results prior to publication. At the time part of this investigation was carried out, one of us (S.F.) was visiting the Department of Physics of Stanford University. He wishes to express his most sincere thanks for the very kind hospitality extended to him and to acknowledge the very illuminating discussions he had on this subject with Professor L. Schiff and Professor R. Hofstadter.

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PION-PION INTERACTION IN THE PHOTOPRODUCTION OF NEUTRAL PIONS WITH POLARIZED γ RAYS

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The presence of a possible pion-pion interaction in a simple resonance state $T=1$, $J=1$ has already been investigated by several authors.[1-6] In this Letter we shall see the effect of this interaction in the case of π^0 photoproduction from the reaction $\gamma+p\to\pi^0+p$ with polarized γ. By using the Cini-Fubini[7] approximate version of the Mandelstam representation, one can demonstrate[4] that in order to take into account the pion-pion interaction in the photoproduction to a reasonable approximation, one should simply add to the expression of the scattering amplitude of CGLN[8] a contribution of the form of the Born term that derives from the graph of Fig. 1. The contribution S_β' (β is the isotopic spin index) to the total S matrix is found to be[6]

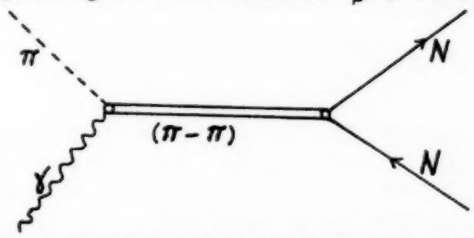


FIG. 1. The first-order perturbation graph which takes into account the π-π interaction. The introduction of the intermediate particle (π-π) simulates the effect of a narrow $J=1$, $T=1$ pion-pion resonance in the intermediate state.

$$S_\beta' = i(2\pi)^4\delta_4(p_1+k-p_2-q) \times \frac{1}{(4k_0\omega E_1E_2)^{1/2}}\bar{u}(p_2)Mu(p_1)\tau_\beta, \quad (1)$$

with

$$M=\frac{8\pi\Lambda}{(p_2-p_1)^2+(4\mu)^2} \times\left[M_D-\frac{\mu_p'-\mu_n}{2M}[M_A(2q\cdot k+\mu^2)-M_\beta]\right], \quad (2)$$

where k, q, p_1, and p_2 are the 4-momenta of the photon, pion, initial nucleon, and final nucleon, respectively; k_0, ω, E_1, and E_2 are the corresponding energies; μ_p' and μ_n are the anomalous magnetic moments of the proton and the neutron; M = nucleon mass; μ = pion mass. The "mass" of the intermediate "particle" is assumed to be $\sim 4\mu$. The constant Λ is proportional to the "strength" of the photon–three-pion interaction and M_A, M_B, M_C, M_D are the invariant terms defined by Chew et al.[8]

In order to carry out the calculation, one simply has to add the coefficients of the invar-

iants appearing in (2) to the corresponding four amplitudes $A^{(0)}$, $B^{(0)}$, $C^{(0)}$, $D^{(0)}$, defined in reference 8.

The π^0 photoproduction with polarized γ seems to be suitable to test the effect of the π-π interaction for two reasons: (a) because of the smallness of the electric dipole term and the absence of the "retardation term"; (b) because the pion-pion interaction acts in opposite directions in the π^0 photoproduction with polarized γ parallel and perpendicular to the production plane, respectively.

The ratio $d\sigma_{\parallel}/d\sigma_{\perp}$ has been studied and found to be strongly affected by the above situation.[9] This does not happen for nonpolarized γ:

$$d\sigma_{\text{unpolarized}} = \tfrac{1}{2}(d\sigma_{\parallel} + d\sigma_{\perp}).$$

Explicitly calculating the ratio $d\sigma_{\parallel}/d\sigma_{\perp}$ as a function of the incident photon energy E_γ in the laboratory system and for one determined angle of production, $\theta_{\text{c.m.}}$, in the center-of-mass system, we obtain the two curves of Fig. 2(a). Curve A is obtained when one takes into account the effect of the pion-pion interaction and curve B when one does not take it into account; the angle $\theta_{\text{c.m.}}$ is 90°. There is almost the same difference between A and B when $\theta_{\text{c.m.}} = 45°$ and $\theta_{\text{c.m.}} = 135°$.

In both cases we have taken the small p-wave phase shifts directly from experimental data[10] without using the formula for the effective range, since this, as is well known, is in contradiction with the experimental results. This choice is in agreement with the theoretical curves of Bowcock et al.[3] that take into account the pion-pion interaction.

The $N^{(+)}$ associated with the electric dipole amplitude[8] is set equal to zero.

For the s-wave phase shift the values used are $\delta_1 = 0.173q$ and $\delta_3 = -0.110q$.

Finally, in the effective-range formula for the $p_{3,3}$-wave phase shifts, we have used $f^2 = 0.081$ and $\omega_{\text{res}} = 2.1$.

As for the term of the pion-pion interaction, we take $\Lambda = 1.46$ ef since this value is in agreement with experimental data on the ratio $d\sigma^+/d\sigma^-$ of the π^+ and π^- photoproduction cross sections.[6] On the other hand, in this work the value of the constant a, introduced in reference 6 to take into account the high-energy contribution of the dispersion integrals, is put equal to zero because it has no influence in determining the ratio $d\sigma_{\parallel}/d\sigma_{\perp}$.

We have also investigated how these results depend on the choice of the different parameters: The variation of $N^{(+)}$ in the interval -0.1 to +0.1 produces only a shift of the curves of a few percent; of the same order are the variations when we change the s-wave phase shifts compatibly with experimental data.

On the contrary, the curves are relatively sensitive to the values of small p-wave phase shifts. In Fig. 2(b) are drawn the areas where the two curves vary by changing these phase shifts within the experimental errors. The shaded area (A_1) belongs to the curve A and (B_1) to B. Notice that the two areas are not superposed and the behavior of the two curves, which is quite different in the two cases, is conserved.

In Fig. 2(c) we show the dependence of the curves on ω_{res}: By changing the value $\omega_{\text{res}} = 1.9$ to 2.2, the point of intersection is displaced to the right. The curves are rather insensitive to the value of f^2 in the interval 0.08 to 0.088. Finally, we emphasize that the curve A remains nearly the same if we set the value $t_\gamma = 22.4\mu^2$ in accord with reference 3, instead of $t_\gamma = 16\mu^2$.

Thus, upon changing all these parameters, the two curves A and B, although altered, conserve their different behaviors.

It follows that if accurate measurements of $d\sigma_{\parallel}/d\sigma_{\perp}$ agree with values characterized by the area A_1, then, in fact, the pion-pion interaction is strong and a value of $\Lambda \cong 1.46$ is favored. On the other hand, if the values give the behavior that characterizes the area B_1, we want to stress that it is possible that the π-π interaction exists but is so weak (Λ small) that it cannot visibly separate the two curves; thus in this case it is impossible to draw any conclusion.

Another remark we would like to make is that, since the "retardation term" is absent, we may also obtain information about the behavior of the π-π interaction by studying the values of the relation

$$PA = \left(\frac{R-1}{R+1}\right)(\sigma_{\text{unpolarized}}/\sin^2\theta), \qquad (3)$$

defined by Taylor and Mozley[9] and by Smith and Mozley,[11] where P is the polarization and R is the meson production asymmetry. Equation (3) presupposes that if only S and P waves are present, and if the π-π interaction does not exist, then measurements at different angles will give the same value of PA.

The predictions for the values of A at two different energies of the incident photon and three

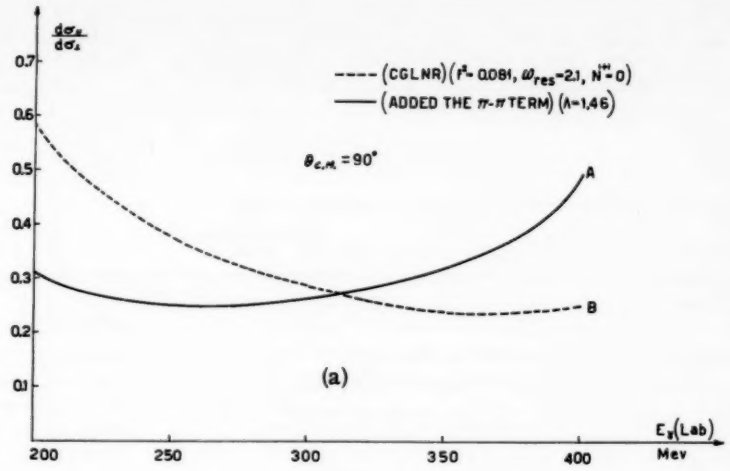
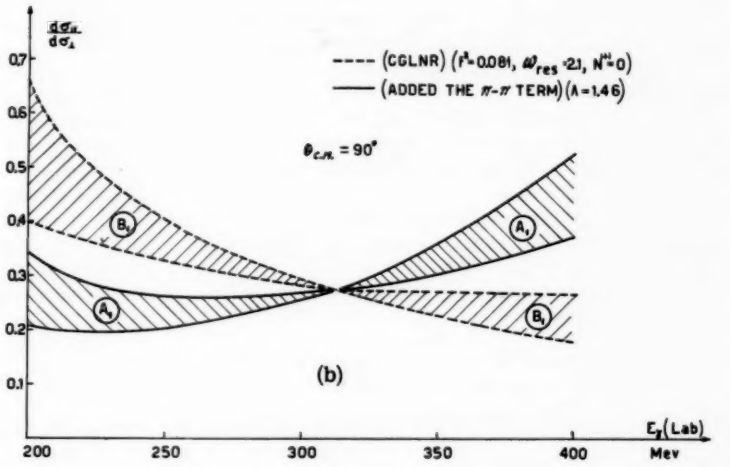
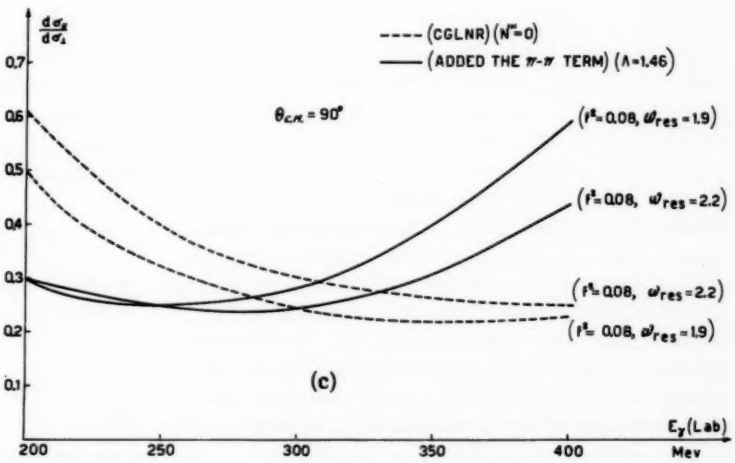


FIG. 2. (a) The ratio $d\sigma_\parallel/d\sigma_\perp$, (1) taking into account the effect of the π-π interaction (curve *A*); (2) omitting the π-π interaction (curve *B*).



(b) The areas where the curves *A* and *B* vary by changing the *P*-wave phase shifts within the experimental errors.



(c) The curves *A* and *B* for two values of ω_{res}: 1.9 and 2.2 ($f^2 = 0.08$).

Table I. Values of A (in units of 10^{-30} cm^2/sr).

	$\theta_{c.m.}$ \ k_0	200 Mev	250 Mev
CGLN terms only	45°	0.5102	4.9097
	90°	0.5117	4.9604
	135°	0.5133	4.9901
With π-π term added	45°	1.3604	8.3041
	90°	1.2485	7.6543
	135°	1.1524	7.1621

angles are summarized in Table I.

We are indebted to Professor M. Cini for many suggestions and his constant helpful advice.

*With a scholarship of the Greek Atomic Energy Commission.

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CONSEQUENCE OF THE CROSSING SYMMETRY FOR A CLASS OF π-π SCATTERING DIAGRAMS*

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In the last few years a considerable effort has been devoted to the study of the π-π interaction with the hope of reproducing the p-wave resonance introduced by Frazer and Fulco to fit the electromagnetic structure of the nucleon. Although the $(\Phi_\alpha\Phi_\alpha)^2$ interaction has not been thoroughly investigated, it has been shown that some of the desired properties of the π-π system can be understood by the introduction of additional intermediate mesons or new interactions.[1] The latter calculations are distinguished by the following approximation; the diagrams of importance are those in which the initial and final mesons are absorbed and created in pairs. Without reference to any specific interaction we would like to exploit a further consequence of this approximation which also amounts to requiring that the π-π scattering amplitude for $I=1$ is the following function of c.m. momentum q and angle θ:

$$A_1 = f[q^2(1-\cos\theta)] - f[q^2(1+\cos\theta)]. \quad (1)$$

As a consequence of the crossing symmetry[2] and Eq. (1), it follows that

$$\tfrac{1}{3}(2A_0 - 5A_2) = 2f[-2(1+q^2)] - f[q^2(1+\cos\theta)] - f[q^2(1-\cos\theta)], \quad (2)$$

where A_0 and A_2 are the scattering amplitudes for $I=0$, 2, respectively. Let the p-wave amplitude be

$$\int_{-1}^{+1} A_1 \cos\theta \, d\cos\theta = G(q^2), \tag{3}$$

where

$$G(q^2) = -16\pi(\omega/q)e^{i\delta_1}\sin\delta_1.$$

Equation (3) can be converted into a differential equation for f which yields

$$f(x) = -\tfrac{1}{2}\int_0^x \frac{G(y)}{y}\,dy - \tfrac{3}{4}G(x) - \tfrac{1}{4}x\,G'(x). \tag{4}$$

It is clear that the determination of the s phase requires a continuation of the p phase shifts to $q^2<0$. Using an effective-range approximation for the phase shifts, Eq. (4) yields

$$2a_0 - 5a_2 = 18a_1. \tag{5}$$

Using the p-wave scattering lengths of Frazer and Fulco[3] and Bowcock et al.,[4] respectively, in Eq. (5) yields[5]

$$2a_0 - 5a_2 = 4.8\mu^{-1},\ 0.73\mu^{-1}. \tag{6}$$

In comparison with this, Sawyer and Wali[6] obtain $0.9\mu^{-1}$ and Schnitzer[7] gets $\sim 0.005\mu^{-1}$. Combining the $I=0$ s-wave scattering length $a_0 = 1\mu^{-1}$ determined by Efremov et al.[8] and Ishida et al.[9] with the value of a_2 given in reference 6, we obtain $2a_0 - 5a_2 = 4.5\mu^{-1}$; and combining the Khuri and Treiman[10] value of $a_2 - a_0 = 0.8\mu^{-1}$ with that of references 8 and 9, we obtain $2a_0 - 5a_2 = -7\mu^{-1}$. Based on the best p phase shift of reference 5 and Eq. (6), we conclude that a positive and large π-π scattering length in the $I=0$ state is inconsistent with experiment.

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*Work supported in part by the U. S. Atomic Energy Commission.

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K^- ABSORPTION AND THE $K\Sigma N$ PARITY

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The angular distributions for the three processes,

$$K^- + p \to \pi^- + \Sigma^+, \tag{I}$$

$$K^- + p \to \pi^+ + \Sigma^-, \tag{II}$$

$$K^- + p \to K^- + p, \tag{III}$$

at 400-Mev/c lab-system K momentum, appear to contain large $\cos^2\theta$ terms.[1] The experimental data are scanty, but roughly 3/4 of the events for each of the three processes corresponds to a center-of-mass scattering angle in the regions $|\cos\theta| > \frac{1}{2}$.[1] It is generally assumed that these large $\cos^2\theta$ terms (which we shall refer to as the 400-Mev/c anomaly) result from interactions in the $P_{3/2}$ state of the $K^- - p$ system. The purposes of this note are to point out that if the $K\Sigma N$

parity is even, the anomaly may occur in the $D_{3/2}$ $K^- - p$ state, and to suggest a Σ^+ polarization measurement that would clarify the experimental situation.

We consider two different possibilities concerning the intrinsic parities $\mathcal{P}$ of the particle pairs, $\bar{K}N$, $\pi\Lambda$, and $\pi\Sigma$:

$$\mathcal{P}(\bar{K}N) = \mathcal{P}(\pi\Lambda) = \mathcal{P}(\pi\Sigma), \quad \text{(A)}$$

$$\mathcal{P}(\bar{K}N) = \mathcal{P}(\pi\Lambda) = -\mathcal{P}(\pi\Sigma). \quad \text{(B)}$$

In both parity cases we denote the partial wave amplitude for any of the three processes by the symbol $t_{a,b}$, where a represents the total angular momentum, and b the orbital angular momentum in the initial $(K^- + p)$ state. If parity assignment (A) is correct, it is highly probable that the large $\cos^2\theta$ terms result from the amplitudes $t_{3/2,1}$, as it is difficult to construct a reasonable model in which D- or F-wave interactions are strong while P-wave interactions are weak at such a low energy. Furthermore, an $I=1$, $P_{3/2}$ pion-hyperon resonance seems likely in this case.[2] On the other hand, if parity assignment (B) is correct, $P_{3/2}$ $\pi\Sigma$ pairs result from $D_{3/2}$ $\bar{K}N$ pairs, and vice versa. Since the center-of-mass momenta in the $\pi\Sigma$ and $\bar{K}N$ states are comparable at 400-Mev/c bombarding momentum, one cannot predict with any confidence which of the two amplitudes $t_{3/2,1}$ or $t_{3/2,2}$ is more likely to be large for $\Sigma + \pi$ production. In either case, the requirements of unitarity and analyticity could lead to a large value of the corresponding $K^- - p$ elastic scattering amplitude.

It is instructive to consider the theoretical possibility of a low-energy $j = 3/2$ resonance in either the $\pi\Sigma$ or $\bar{K}N$ state under parity assumption (B), assuming that the pion-baryon interactions are of the conventional, renormalizable, Yukawa type. The Born approximation terms resulting from the pseudoscalar $\pi\Sigma\Sigma$ and $KN\Lambda$ interactions may contribute large pole terms to some of the P-wave amplitudes. If certain small recoil terms are neglected, the Born approximation vanishes for $P_{3/2}$ $\bar{K}$-N elastic scattering.[3] On the other hand, the pole terms for $P_{3/2}$ π-Σ elastic scattering are $\frac{2}{3}f_{\pi\Sigma\Sigma}{}^2/(\omega\mu^2)$ and $-\frac{1}{3}f_{\pi\Sigma\Sigma}{}^2/(\omega\mu^2)$ for the states of isotopic spins 1 and 0, respectively.[4] [The corresponding pole term for $I=3/2$, $P_{3/2}$, π-N scattering is $\frac{4}{3}f_{\pi NN}{}^2/(\omega\mu^2)$.] The positive, $I=1$ pole term might lead to a π-Σ resonance,[4] and hence to large values of the $\bar{K}$-N absorption and scattering amplitudes $t_{3/2,2}$. It is not known whether or not the 400-Mev/c anomaly results from a pion-hyperon resonance, but if it does, the observed small value (~0.4) for the $(K^- + p \to \pi^0 + \Lambda)/(K^- + p \to \pi^\mp + \Sigma^\pm)$ branching ratio fits better with parity assignment (B) than with (A). [If $\mathcal{P}(\Sigma) = \mathcal{P}(\Lambda)$, one expects the $\pi + \Lambda$ component of the $I=1$, $P_{3/2}$ resonance to be stronger than the $\pi + \Sigma$ component.[2,3]]

One can hope to determine the parity of the amplitude responsible for the 400-Mev/c anomaly by observing interference with the amplitude $t_{1/2,0}$, known to be large at lower energies for all three processes under discussion. We assume that the angular momentum of the anomaly is 3/2. The interference between $t_{1/2,0}$ and the $j = \frac{3}{2}$ amplitudes in the differential cross section for any of the three processes is proportional to the quantity

$$2\,\mathrm{Re}[2t_{1/2,0}t_{3/2,1}{}^* \cos\theta \pm t_{1/2,0}t_{3/2,2}{}^*(3\cos^2\theta - 1)],$$

where the lower sign applies only for $\pi + \Sigma$ production under parity assumption (B). Since it is likely that the amplitudes $t_{1/2,0}$ for the three processes are still appreciable at 400 Mev/c, the approximate symmetry of the 400-Mev/c cross sections around 90° is favorable evidence that the $K^- - p$ orbital parity of the anomaly is even. This evidence is weak, however, since the $j = \frac{3}{2}$ amplitudes may be nearly 90° out of phase with the amplitudes $t_{1/2,0}$. The existence of an appreciable $\cos\theta$ term in the $K^- - p$ elastic scattering angular distribution[1] at 300 Mev/c might be considered as evidence that the amplitude $t_{3/2,1}$ is growing with energy. This $\cos\theta$ term is not large, however, and may result from a small admixture of the amplitude $t_{1/2,1}$. We conclude that there is no strong experimental evidence concerning the parity of the 400-Mev/c anomaly.

Further information may be obtained by measuring the up-down asymmetry of the proton emitted in the decay of the Σ^+ of reaction (I), since this asymmetry measures the Σ^+ polarization. The interference between $t_{1/2,0}$ and the $j = \frac{3}{2}$ amplitudes in the polarization intensity (percentage polarization in the direction of $\vec{k}_i \times \vec{k}_f$, multiplied by the differential cross section) is proportional to

$$2\,\mathrm{Im}[\pm t_{1/2,0}t_{3/2,1}{}^* - 3t_{1/2,0}t_{3/2,2}{}^* \cos\theta]\sin\theta,$$

where the lower sign again applies only if the intrinsic parities of the initial and final particle pairs are opposite. The parity of the 400-Mev/c anomaly in $\Sigma^+ + \pi^-$ production may be determined definitely from angular distribution and polarization intensity measurements in the range of

K lab-momentum 150-400 Mev/c. In fact, the detection of either a large $\sin\theta$ or $\cos\theta\sin\theta$ dependence of the polarization intensity at 400 Mev/c would provide a strong clue to this parity, since it is likely that the amplitude $t_{1/2,0}$ is still larger than $t_{1/2,1}$ at this energy.

We conclude that the appropriate polarization measurements should be made in order that the parity of the 400-Mev/c anomaly in reaction (I) may be determined. If the $K^- - p$ orbital angular momentum involved is even, it is unlikely that the intrinsic parities of the $\bar{K}N$ and $\pi\Sigma$ states are equal.

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ODD $\Lambda\Sigma$ PARITY AND THE NATURE OF THE $\pi\Lambda\Sigma$ COUPLING*

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In various symmetry models of strong interactions it has often been assumed that Λ and Σ belong to the same "supermultiplet" in some approximation.[1-3] Such a picture necessarily requires that the relative $\Lambda\Sigma$ parity be even. The purpose of the present Letter is twofold. We first summarize the recent experimental developments which are indicative of odd $\Lambda\Sigma$ parity. Secondly, we point out some unusual features of the scalar $\pi\Lambda\Sigma$ coupling; in particular we show that the scalar coupling constant is "calculable" from m_π, m_Λ, and m_Σ, and that the scalar coupling constant we calculate is in good agreement with that deduced from hypernuclear physics.

We wish to point out that, although even $\Lambda\Sigma$ parity has been tacitly assumed by many theoreticians, the available experimental data are suggestive of odd, rather than even, $\Lambda\Sigma$ parity. We can see this in the following eightfold way:

(1) According to recent Cornell data on associated photoproduction,[4] the angular distribution of

$$\gamma + p \to \Sigma^0 + K^+ \qquad (1)$$

at $E_\gamma = 1140$ Mev (threshold $E_\gamma = 1040$ Mev) seems anisotropic, and is reminiscent of a retarded $\sin^2\theta$ distribution, whereas at comparable K momenta the angular distribution for

$$\gamma + p \to \Lambda^0 + K^+ \qquad (2)$$

shows practically no structure. This feature agrees with the conventional view that the photoproduction of a charged meson near threshold takes place via the electric dipole absorption of the incident photon, only if K is pseudoscalar with respect to Λ (for which there is evidence from K^--He experiments[5]) but scalar with respect to Σ.[6]

(2) If a Taylor-Moravcsik type extrapolation analysis[7] is made for reaction (1), the available data strongly favor even $K\Sigma$ parity provided that only s waves and s-p interference are significant for the contributions other than the one-K exchange term (meson current term), or, what amounts to the same thing, provided that $(1-\beta_K\cos\theta)^2(d\sigma/d\Omega)$ can be correctly extrapolated to $\cos\theta = \beta_K^{-1}$ by the use of a third-order polynomial with one constraint.[8] (The de Broglie wavelength of the K particle in the c.m. system is as large as 1.0×10^{-13} cm so that our cubic extrapolation may be justified.) Since there is some evidence for odd $K\Lambda$ parity elsewhere,[5,7] we see that odd $\Lambda\Sigma$ parity is favored.

(3) In the angular distribution for the reaction

$$\pi^- + p \to \Lambda^0 + K^0 \qquad (3)$$

at the ΣK threshold, an anomaly in the $\cos^3\theta$ term has been reported by Schwartz and collaborators.[9] If f waves are relatively unimportant for the final ΛK system at $p_K^{(\mathrm{c.m.})} \approx 230$ Mev/c ($\lambda_K \approx 0.85\times10^{-13}$ cm), then this anomaly should be attributed to p-d interference, which in turn

implies that the $\Lambda\Sigma$ parity is odd.[10]

(4) In associated production of Σ particles in πp collisions no striking backward peaking of Σ's similar to the backward peaking of Λ's has been observed at low energies ($E_\pi^{(KE)} \lesssim 1.1$ Bev), although there is evidence for a backward peaking at higher energies. As already pointed out by Bég, Bernstein, and De Celles,[11] this feature can be nicely explained if the backward peaking of the hyperon is due to the pole arising from the exchange of a K^* (880-Mev $K\pi$ resonance) which is scalar with respect to Λ but pseudoscalar with respect to Σ.

(5) Preliminary indications[12] seem to favor the spin $\frac{1}{2}$ assignment for Y^* (the $T=1$ $\pi\Lambda$ resonance discovered by Alston et al.[13]). If the relative $K\Lambda$ parity is odd,[5] and if $Y^* \to \pi + \Lambda$ via the $s_{1/2}$ channel, for which there is some preliminary evidence,[12] then Y^* has the same quantum numbers as the low-energy $\bar{K}N$ system, which means that Y^* is a resonance of the type discussed by Dalitz and Tuan.[14] In such a case the observed small $\pi\Sigma/\pi\Lambda$ ratio for Y^* decay seems to favor odd $\Lambda\Sigma$ parity, as recently emphasized by Ross and Shaw[15] and by Dalitz[16] (unless physically unlikely values of effective-range parameters are assumed).

(6) Regardless of the validity of the global symmetry model, a $T=1$, $J=\frac{3}{2}$ p-wave pion-hyperon resonance seems to be expected for a wide range of values of $G_{\pi\Lambda\Sigma}{}^2/4\pi$ and $G_{\pi\Sigma\Sigma}{}^2/4\pi$ as long as the relative $\Lambda\Sigma$ parity is even.[17] The nonexistence of a $T=1$, $J=\frac{3}{2}$ resonance might be taken as an argument against even $\Lambda\Sigma$ parity.

(7) In the even $\Lambda\Sigma$ parity case it is reasonable to expect, in addition, a $T=2$, $J=\frac{3}{2}$ p-wave $\pi\Sigma$ resonance, denoted by Z^*.[17] As pointed out by Lee and Yang,[18] if we take the global symmetry model seriously, this hypothetical $T=2$ state may be at 1539 Mev with a width about twice that of the πN three-three resonance. No evidence for Z^{*++} ($\to \Sigma^+ + \pi^+$) has been reported in the reaction

$$\bar{K}^0 + p \to \Sigma^+ + \pi^+ + \pi^-, \tag{4}$$

studied at $p_K \sim (1.0 \pm 0.1)$ Bev/c by the Yale-Brookhaven bubble chamber group,[19] nor is there any indication for a $\pi\Sigma$ resonance in K^-p experiments.[12,13] [The threshold for Z^* ($\to \Sigma + \pi$) production in association with a π in $\bar{K}N$ collisions is at 750 Mev/c.]

(8) Barshay and Schwartz[20] have pointed out that the remarkable Gell-Mann–Rosenfeld triangle in Σ decay can be understood in terms of a single parameter if the Σ hyperon is a bound $\pi\Lambda$ system. Unless the spin-orbit force between π and Λ is unexpectedly strong, the bound $\pi\Lambda$ system is likely to be $s_{1/2}$.

We admit that none of these indications offers conclusive evidence against even $\Lambda\Sigma$ parity. But they do suggest that speculations on the odd $\Lambda\Sigma$ parity situation might be of more than academic interest.

We now turn our attention to the nature of the $\pi\Lambda\Sigma$ coupling, which, in the following discussion, is assumed to be of the scalar type. It has been known for some time that, if one of the particles in a trilinear coupling can be regarded as a bound system of the other two in the sense of dispersion theory,

$$m_1 < m_2 + m_3, \qquad m_1{}^2 > m_2{}^2 + m_3{}^2, \tag{5}$$

then so-called "structure effects" are expected to be important.[21] In particular, the coupling constant is determined by the asymptotic form of the bound-state wave function. Considerations along these lines have been made in connection with the "neutron-proton-deuteron coupling"[22] and also with a Fermi-Yang type model of the pion.[23]

Although one can discuss the $\pi\Lambda\Sigma$ coupling using the language of dispersion theory, we shall adopt here a more "low-brow" language. In a purely phenomenological discussion, the bound-state picture for Σ can be used as long as the Σ is coupled strongly to the $\pi\Lambda$ system, regardless of whether or not one subscribes to the bound-state picture in a literal sense. Let us consider the asymptotic wave function for the bound $\pi\Lambda$ system,

$$\psi \sim [\exp(-r/r_\Sigma)]/r, \qquad r_\Sigma = (2\mu_\pi |E_B|)^{-1/2}, \tag{6}$$

where

$$E_B = m_\Sigma - m_\pi - m_\Lambda = -60 \text{ Mev}, \qquad \mu_\pi = m_\pi m_\Lambda (m_\pi + m_\Lambda)^{-1} = 120 \text{ Mev}.$$

Physically speaking, r_Σ is the "radius" of the Σ hyperon analogous to the deuteron radius. Numerically,

$$r_\Sigma = 1.6 \times 10^{-13} \text{ cm}.$$

(A fully relativistic treatment leads to essentially the same value.) The surprisingly large value of this number is the starting point of our investigation.

If the range of the $\pi\Lambda$ force is much shorter than the "radius" of the bound $\pi\Lambda$ system, the logarithmic derivative of the wave function evaluated at some radius just outside the range of the $\pi\Lambda$ force does not vary appreciably as the energy is lowered from the zero kinetic energy to the observed binding energy of -60 Mev. It then becomes possible to equate the scattering length a to the "radius" r_Σ:

$$-a = r_\Sigma = (2\mu_\pi |E_B|)^{-1/2}. \tag{7}$$

This relation is simply the well-known relation between the binding energy and the scattering length in the zero-range approximation.

Generally speaking, the range of the $\pi\Lambda$ force is expected to be roughly equal to or less than $(2m_\pi)^{-1}$. Since the pion and the Λ hyperon cannot exchange a pair of pions having the same quantum numbers as the Frazer-Fulco resonance ($T=1$, $J=1$, even G parity), it is probably legitimate to ignore pion-pion interactions. The $\pi\Lambda$ force may then be as short-ranged as the force associated with the so-called "baryon exchange" diagram whose range $\approx (m_\pi m_\Sigma)^{-1/2} = 0.4\times10^{-13}$ cm is much smaller than the "radius" of the Σ hyperon. Physically speaking, this means that the Σ hyperon has a relatively "open" structure; the pion and the Λ hyperon spend most of their time outside the range of the interaction.[24]

We now calculate the scalar coupling constant $G_{\pi\Lambda\Sigma}{}^2/4\pi$. This can be carried out either by noting that the normalization constant for the asymptotic bound-state wave function is related to the "sticking probability" for $\pi+\Lambda\to\Sigma$ or by comparing (7) with the expression for the scattering length computed under the assumption that the Σ pole dominates in low-energy $\pi\Lambda$ scattering. We obtain

$$\frac{G_{\pi\Delta\Sigma}{}^2}{4\pi} = \frac{m_\pi}{\mu_\pi}\left(\frac{|E_B|}{2\mu_\pi}\right)^{1/2}$$

$$= \left(\frac{m_\pi+m_\Lambda}{m_\Lambda}\right)\left(\frac{(m_\pi+m_\Lambda-m_\Sigma)(m_\pi+m_\Lambda)}{m_\pi m_\Lambda}\right)^{1/2}$$

$$\approx 0.6. \tag{8}$$

Of course, this formula is exact only in the zero-range approximation. If the effective range for $\pi\Lambda$ scattering is 0.3×10^{-13} cm, then the value of the coupling constant gets increased by about 15%.

That we can calculate the coupling constant from the mass spectrum alone is the most interesting feature of the scalar-type $\pi\Lambda\Sigma$ coupling. The physical origin of this remarkable fact is clear; although we do not know the detailed dynamics of the short-range interaction between π and Λ, as far as its large-scale manifestations such as the scattering length and the binding energy are concerned, only one parameter, depth times (range)^2 in potential theory, is relevant, which means that E_B and $G_{\pi\Lambda\Sigma}{}^2/4\pi$ are not independent of each other.[25]

Up to now the only other information on the $\pi\Lambda\Sigma$ coupling comes from the ΛN potential fitted to the binding energies of various hypernuclei. Ferrari and Fonda[26] have shown that if the $\Lambda\Sigma$ parity and the $K\Lambda$ parity are both odd, the binding energy of ${}_\Lambda\mathrm{He}^5$ together with the requirement that the two-body ΛN system be unbound leads to the coupling constant combination

$$G_{\pi\Lambda\Sigma}{}^2/4\pi = 0.5, \quad \text{(scalar coupling)}$$

$$G_{K\Sigma N}{}^2/4\pi \gtrsim 0.8, \quad \text{(scalar coupling)}$$

$$3 > G_{K\Lambda N}{}^2/4\pi > 0, \quad \text{(pseudoscalar coupling)}.$$

So our result on the $\pi\Lambda\Sigma$ coupling constant is in good agreement with the calculations of Ferrari and Fonda.

It is hoped that the present note will prompt experimentalists to concentrate their efforts on experiments that have bearings on an unambiguous determination of the relative parity. Search for possible correlation effects in

$$\Sigma^0 \to \Lambda^0 + e^+ + e^- \tag{9}$$

seems particularly promising.[27]

To conclude, if there is one observable on which the future of strange particle physics depends, it is the relative $\Lambda\Sigma$ parity.[28]

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*Work supported by the U. S. Atomic Energy Commission.

[1] J. Schwinger, Ann. Phys. 2, 407 (1957); M. Gell-Mann, Phys. Rev. 106, 1296 (1957). It appears that the global symmetry model has recently been disowned by Gell-Mann.[3]

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[3]M. Gell-Mann, "The Eightfold Way: A Theory of Strong Interaction Symmetry" (unpublished).

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[5]G. Puppi, Proceedings of the 1960 Annual International Conference on High-Energy Physics (Interscience Publishers, Inc., New York, 1960), p. 419.

[6]In the even $\Lambda\Sigma$ parity case, it may be difficult to explain why the magnetic dipole absorption is anomalously strong for Σ^0 production but not for Λ^0 production.

[7]M. J. Moravcsik, Phys. Rev. Letters 2, 352 (1959).

[8]A detailed account on this point will be given elsewhere. Preliminary investigations made by one of us (J.J.S.) have led to the scalar constant $G_{K\Sigma N}^2/4\pi \sim 0.6$. Since there are only three experimental points available, extrapolations with higher order polynomials are not feasible at the present moment.

[9]M. Schwartz, in Proceedings of the Conference on Strong Interactions, Berkeley, California, 1960 [Revs. Modern Phys. (to be published)].

[10]The argument goes as follows: It has been established experimentally by the Berkeley hydrogen bubble chamber group that the energy and angular dependence of $\pi^- + p \rightarrow \Sigma^- + K^+$ very near threshold is typical of the s-state excitation. So the cusp state of the ΛK system must be either $s_{1/2}$ (even $\Lambda\Sigma$) or $p_{1/2}$ (odd $\Lambda\Sigma$). In the $s_{1/2}$ case, the observed $\cos^3\theta$ anomaly would be possible only via s-f interference.

[11]M. A. Baqi Bég, J. Bernstein, and P. C. De Celles, Brookhaven National Laboratory Internal Report PD-23, 1961 (unpublished).

[12]See, e.g., reports by R. K. Adair, M. Ferro-Luzzi, and M. Alston, in Proceedings of the Conference on Strong Interactions, Berkeley, California, 1960 [Revs. Modern Phys. (to be published)]. Also M. M. Block (private communication).

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[15]M. Ross and G. Shaw, Phys. Rev. Letters 5, 578 (1960).

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[22]M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. 2, 226 (1957); G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[23]Y. Nambu, in Proceedings of the Midwest Conference on Theoretical Physics (Purdue University, Lafayette, Indiana, 1960).

[24]It is of interest to compare this situation with Y^* assuming that Y^* is a bound $\bar{K}N$ system in the s state as suggested by various people.[15,16] In the Y^* case the radius turns out to be 1.1×10^{-13} cm, and the range of the $\bar{K}N$ force might be as large as $(2.2 m_\pi)^{-1} \sim 0.65 \times 10^{-13}$ cm, particularly if the anomalous state seen in p-d collisions by A. Abashian, N. E. Booth, and K. M. Crowe [Phys. Rev. Letters 5, 258 (1960)] turns out to be a neutral vector particle coupled to the hypercharged current [J. J. Sakurai, Ann. Phys. 11, 1 (1960); see also Gell-Mann[3]]. Incidentally, if the Σ is regarded as a bound state of a $\bar{K}$ and an N as in Goldhaber's model [M. Goldhaber, Phys. Rev. 101, 433 (1956)], the Σ "radius" under this assumption turns out to be 0.6×10^{-13} cm so that no reliable information on the $K\Sigma N$ coupling can be obtained from analogous considerations.

[25]It has been suggested that in the odd $\Lambda\Sigma$ parity case one may still equate, in the bare Lagrangian, the pseudoscalar $\pi\Sigma\Sigma$ coupling constant to the scalar $\pi\Lambda\Sigma$ coupling constant [S. Barshay, Phys. Rev. Letters 1, 97 (1958); F. Gürsey, Phys. Rev. Letters 1, 98 (1958); R. E. Behrends, Nuovo cimento 11, 424 (1959)]. However, (8) tells us that even if the unrenormalized pseudoscalar constant and the unrenormalized scalar constant did satisfy a certain relation, there would be no reason whatsoever to expect a similar relation for the corresponding renormalized constants.

[26]F. Ferrari and L. Fonda, Nuovo cimento 9, 842 (1958). These authors use a repulsive core of radius $(3m_\pi)^{-1}$ at short distances. Such a core in the ΛN case may arise naturally if there exists a heavy neutral vector particle coupled to the conserved baryon current.

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[28]It appears that the original vector theory of strong interactions proposed by one of us (J.J.S.[24]), but not Gell-Mann's "eightfold way,"[3] is the only "verifiable" symmetry model which can accommodate (but does not require) odd $\Lambda\Sigma$ parity.

LAW OF CONSERVATION OF MUONS*

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The apparent absence of muon-electron transitions without neutrinos, such as $\mu \to e + \gamma$, $\mu \to 3e$, and $\mu^- + p \to e^- + p$, leads one to suspect that there is a new conservation law forbidding them. Calculations[1] of the rate of such processes, assuming no such law exists, have indicated that it is hard to understand their absence in an intermediate boson theory of weak interactions. Even if there is no intermediate boson, the decay $\mu \to e + \nu_1 + \bar{\nu}_2$ with $\nu_1 = \nu_2$ would lead to these processes in some order of perturbation theory, and arguments have been given[2] which indicate that any field theory of weak interactions may predict unacceptably large rates for these processes in the absence of a selection rule.

If we assume that $\mu^- - e^-$ transitions are forbidden by a selection rule, the nature of the selection rule remains an open question. It has been suggested[3] that an additive quantum number exists which is always conserved, and which[4] is +1 for μ^- and zero for e^-. In order to make this consistent with known weak interactions, it is necessary to assume that there are two neutrinos, which are distinguished by their value of this quantum number. The conservation law forbids all reactions in which any nonzero number of muons change into electrons, without neutrinos.

This assumption of an additive conservation law is not the only possibility. All of the "missing reactions" involve odd numbers of muons and electrons. It is therefore possible to forbid them by a multiplicative conservation law. By this it is meant that there is a quantity we shall call "muon parity" which is -1 for the muon and its neutrino, and +1 for electrons and all other known particles.[5] The muon parity of a system of particles is the product of its values for the individual particles, and is to be universally conserved. The possibility of multiplicative conservation laws has been known for some time,[6] although no law precisely of this type is known to exist at present.

There are certain theoretical arguments in favor of a multiplicative conservation law for muons and electrons. It has recently been shown[7,8] that the symmetry in the properties of muon and electron, as well as their different mass, can be summarized by the invariance of the laws of nature under permutation of two primitive leptons (say e' and μ'). If the e' and μ' can make transitions into each other, they will not be observed as particles, but instead certain linear combinations; $e = (\mu' + e')/\sqrt{2}$, $\mu = (\mu' - e')/\sqrt{2}$, which would be stable in the absence of weak interactions, will be the observed electron and muon, and will necessarily have different mass. Invariance under the permutation symmetry $\mu' \longleftrightarrow e'$ implies invariance under the transformation $e \to e$, $\mu \to -\mu$. It is shown in reference 8 that the extension of the permutation symmetry to weak interactions requires the existence of two neutrinos ν_e, ν_μ, which also transform as $\nu_e \to +\nu_e$, $\nu_\mu \to -\nu_\mu$. This argument therefore leads directly to a multiplicative conservation law of "muon parity." If no particular model is assumed for the weak interactions, no stronger, additive, conservation law is implied. Of course, we cannot rule out the possibility that there is an additive quantum number, which would imply a stronger selection rule than the multiplicative law. This was indeed the case in the specific Lagrangian models studied in references 7 and 8. However, we believe it is worthwhile to consider the consequences of the multiplicative symmetry by itself, and to perform experiments to distinguish between the additive and multiplicative conservation laws.

The two conservation laws both forbid $\mu \to e + \gamma$, $\mu \to 3e$, and $\mu + p \to e + p$. They also imply that two different neutrinos are emitted in μ decay. However, the additive law implies that the μ^+ can only decay by $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$, while the multiplicative law would also allow $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_e$. If the neutrinos from μ decay can be used to induce inverse transitions, the latter possibility could be tested by looking for μ^- produced by neutrinos from μ^+ decay, which is forbidden, by lepton conservation, if only $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$.

If one considers systems with only muons and electrons, the selection rule implied by conservation of muon parity is

$$(-1)^{n_\mu(\text{initial state})} = (-1)^{n_\mu(\text{final state})},$$

where n_μ is the number of muons of either charge present. One reaction which is allowed by it while forbidden by the additive law is $e^- + e^- \to \mu^- + \mu^-$. The cross section for this in a clashing beam experiment at 10 Bev is probably no greater than 10^{-36} cm^2.

Another reaction allowed by muon parity conservation and forbidden by the additive law is the transformation of muonium ($\mu^+ e^- \equiv M$) into antimuonium ($\mu^- e^+ \equiv \bar{M}$). This reaction was first discussed in a theory with no muon conservation law by Pontecorvo.[9] If an interaction exists which allows $M \leftrightarrow \bar{M}$ transitions with matrix element $\delta/2$, then the vacuum energy eigenstates will not be M and $\bar{M}$, but some linear combinations M_1, M_2 with different energies. Thus a system which is pure muonium at $t=0$ will develop an admixture of antimuonium at a later time. We show in another place[10] that the probability of seeing the system in vacuum decay as antimuonium ($\bar{M} \to$ fast e^- + slow e^+ + neutrinos) rather than as muonium ($M \to$ fast e^+ + slow e^- + neutrinos) is given by

$$P(\bar{M}) = \tfrac{1}{2}\delta^2/(\delta^2 + \Delta^2 + \lambda^2), \qquad (1)$$

where λ is the muon decay rate ($= 0.45\times10^{+6}$ sec^{-1}, or 3×10^{-10} ev) and Δ is any additional splitting of M and $\bar{M}$, say by external electromagnetic fields. We can estimate δ by assuming that the $M \leftrightarrow \bar{M}$ transition is produced by the Fermi-type interaction,

$$H = (C_V/\sqrt{2})\bar{\psi}_\mu \gamma_\lambda (1+\gamma_5)\psi_e \bar{\psi}_\mu \gamma^\lambda (1+\gamma_5)\psi_e + \text{H.c.} \qquad (2)$$

If C_V is the vector β-decay coupling constant, we get, for hyperfine $F=0$ and $F=1$ ground states,

$$\delta = 16 C_V/(\sqrt{2}\pi a^3) = 2.1\times10^{-12} \text{ ev} = 3200 \text{ sec}^{-1}, \qquad (3)$$

so that if $\Delta \ll \lambda$, $P(\bar{M}) \simeq \frac{1}{2}\delta^2/\lambda^2 = 2.6\times10^{-5}$, which would probably be observable. An interaction of this type and magnitude would not have shown up in any previous experiments. An estimate[10] of the probability of seeing an e^- which has gained ≥ 10 Mev from the e^+ in an ordinary μ^+ decay gives a value $< 10^{-10}$.

It can be shown[10] that constant external fields do not contribute to Δ in the $F=0$ ground state. Because of this, the splitting in this state for macroscopic external fields is negligible. Thus an experiment to detect the transition, in which the muonium-antimuonium system is in vacuum for most of the muon lifetime, will, for the $F=0$ state, be governed by the vacuum rate (2.6×10^{-5}), even if fields are present.

On the other hand, an experiment done with muonium which remains in matter would give a much lower transition rate. This case is treated in detail in reference 10. In a solid, the energy shift Δ will be much larger than λ, which reduces the rate by a factor $(\lambda/\Delta)^2$. In a gas, the effect of collisions is to make the amplitudes, for making antimuonium in the periods between collisions, add incoherently. If one starts with muonium, the probability of seeing a μ^- decay or be captured by the nucleus of a gas atom is

$$P(\bar{M}) \simeq \delta^2/(2\lambda\omega_c), \qquad (4)$$

where ω_c is the collision rate. Thus the rate is reduced by $\lambda/\omega_c = 1/N$, where N is the number of collisions, compared to the vacuum rate. In a typical experiment in a gas, $1/N$ might be 10^{-4} to 10^{-6}.

We would finally like to indicate the possible relevance of the intermediate-boson hypothesis for these considerations. If all weak interactions go via intermediate bosons, it is necessary for us to assume the existence of bosons with muon parity of -1. Such bosons are forbidden by the conservation laws to interact linearly with pions, baryons, etc. A neutral boson B^0, identical with its antiparticle, could, by interacting with μe pairs, generate the interaction (2). A charged boson B^+, interacting with the pairs $\mu\nu_e$ and $e\nu_\mu$, would generate the decay[11] $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_e$. If such a charged boson exists, it might be detected in the experiments recently proposed[12] to detect the different bosons (W^+, W^0) which may mediate the known weak interactions. Let us assume that the neutrino-scattering experiments ($\nu + n \to \text{lepton} + p$) indicate the existence of two neutrinos. (The forbidding of $\nu_\mu + n \to e^- + p$ is unaffected by the existence of B^+.) The neutrinos in $\pi^- \to \mu^- + \nu$ decay must be ν_μ. The following process may now occur if muon parity is conserved:

$$\nu_\mu + \text{nucleus} \to e^- + B^+ + \text{nucleus}.$$

The B^+ may now decay only into $\mu^+ + \nu_e$ or $e^+ + \bar{\nu}_\mu$, but not into pions, etc. The production of electrons with such a boson which decays into

μ is forbidden in a theory with an additive conservation law. If there were no muon conservation law at all, then electrons may be produced with a boson which decays into pions. The detection of electrons produced by ν_μ, together with a boson of only leptonic decay modes, would be strong evidence in favor of a multiplicative selection rule.

The authors would like to thank many of their colleagues for very valuable discussions.

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†Alfred P. Sloan Foundation Fellow.

[1]G. Feinberg, Phys. Rev. 110, 1482 (1958); F. J. Ernst, Phys. Rev. Letters 5, 478 (1960).

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[3]K. Nishijima, Phys. Rev. 108, 907 (1957); J. Schwinger, Ann. Phys. 2, 407 (1957); S. Bludman, Nuovo cimento 9, 433 (1958).

[4]The assignment given here of the additive quantum number is conventional. A linear combination of this quantum number with lepton number would give another conserved quantum number whose physical implications are the same.

[5]The assignment of muon parity +1 to baryons, pions, etc., follows from the existence of nonleptonic transitions among such particles, and is unique up to factors $(-1)^{\text{baryon number}}$ or $(-1)^{\text{charge}}$ which have no physical significance.

[6]See, e.g., C. N. Yang and J. Tiomno, Phys. Rev. 79, 495 (1950). The properties of multiplicative conservation laws are discussed by G. Feinberg and S. Weinberg, Nuovo cimento 14, 5711 (1959).

[7]G. Feinberg, P. K. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).

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[9]B. Pontecorvo, Zhur. Eksp. i Teoret. Fiz. 33, 549 (1957) [translation: Soviet Phys. – JETP 6(33), 429 (1958)].

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[11]In this case, in order to make the total μ-decay rate come out to agree approximately with the universal coupling theory, it is necessary to reduce the coupling constant of bosons with leptons by 1/2, compared to the coupling with baryons. This may be contrary to the spirit of the universal coupling theory. However, the ambiguities in the definition of "universality" are well known, and have been emphasized recently by M. Gell-Mann.

[12]M. Schwartz, Phys. Rev. Letters 4, 306 (1960); T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).

SPIN OF THE K'

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Recent experimental work on high-energy K-N collisions at Berkeley has been shown by Alston et al.[1] to suggest the existence of an unstable particle K',[2] formed from a K meson and a pion. They further assign to it a mass ~878 Mev, full width 23 Mev, and isospin 1/2. We shall investigate below, on the basis of the assumption that the pions produced from K-N collisions come entirely from the decay of K', whether K' is a vector or a scalar boson. We find that a vector K' fits the experimental data better than a scalar K'.

The following interactions are considered:

$$K' \text{ with spin 0, } H_I = mG\phi_{K'}\phi_K\phi_\pi; \tag{1}$$

$$K' \text{ with spin 0, } H_I = \frac{G}{m}\frac{\partial\phi_K}{\partial x_\mu}\frac{\partial\phi_\pi}{\partial x_\mu}\phi_{K'}; \tag{2}$$

$$K' \text{ with spin 0, } H_I = \frac{G}{m}\frac{\partial\phi_{K'}}{\partial x_\mu}\frac{\partial\phi_K}{\partial x_\mu}\phi_\pi; \tag{3}$$

$$K' \text{ with spin 0, } H_I = \frac{G}{m}\frac{\partial\phi_\pi}{\partial x_\mu}\frac{\partial\phi_{K'}}{\partial x_\mu}\phi_K; \tag{4}$$

$$K' \text{ with spin 1, } H_I = G\left(\phi_\pi\frac{\partial\phi_K}{\partial x_\mu} - \frac{\partial\phi_\pi}{\partial x_\mu}\phi_K\right)\phi_{K'}{}^{\mu}. \tag{5}$$

For the process $K^- + p \to K'^- + p$, shown in Fig. 1, the total cross section using the first interaction is

$$\sigma_1 = \frac{g^2}{4\pi}\frac{m^2G^2}{4\pi}\frac{\pi}{4M^2|q_1|_L^2}\int\frac{\Delta^2}{(\Delta^2+\mu^2)^2}d\Delta^2,$$

$$\frac{m^2G^2}{4\pi} = \frac{4\lambda m_0^2}{3[(m_0^2 - m^2 - \mu^2)^2 - 4m^2\mu^2]^{1/2}},$$

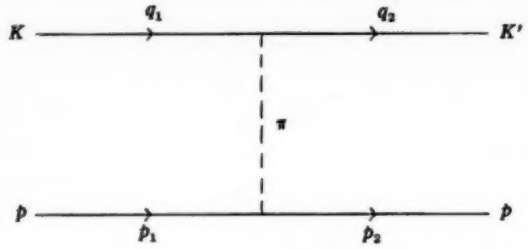

FIG. 1. K' production from K-N collision.

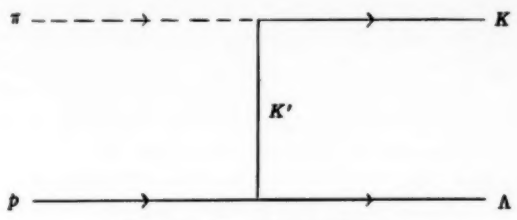

FIG. 2. K' exchange diagram.

where g is the renormalized pion-nucleon coupling constant, m_0 and λ are the mass and width of the unstable particle K', and Δ^2 is the square of the momentum transfer between the nucleons. These equations are similar to Eqs. (5) and (2) of Bég and DeCelles[3] except that in the present work the isotopic spin factors are taken into account. This is important because K' has two possible channels of decay into a K meson and a pion. Similar equations can easily be obtained for cases (2) to (5). Using $m_0 = 878$ Mev, incident pion momentum 1.15 Bev/c, we find

$$\sigma_1(\text{mb}) = 0.0678(\lambda g^2/4\pi)\times 10^{-2},$$
$$\sigma_2(\text{mb}) = 0.1865(\lambda g^2/4\pi)\times 10^{-2},$$
$$\sigma_3(\text{mb}) = 0.1205(\lambda g^2/4\pi)\times 10^{-2},$$
$$\sigma_4(\text{mb}) = 0.01565(\lambda g^2/4\pi)\times 10^{-2},$$
$$\sigma_5(\text{mb}) = 0.854(\lambda g^2/4\pi)\times 10^{-2}, \qquad (6)$$

where λ is in Mev.

However, we are in fact interested only in the process $K^- + p \to \overline{K}^0 + \pi^- + p$ which, as before, is said to proceed according to

$$K^- + p \to K'^- + p \to \overline{K}^0 + \pi^- + p.$$

Hence we have to multiply the results (6) by an isotopic spin factor 2/3. Taking $g^2/4\pi = 14.5$, $\lambda = 23$ Mev $\pm 20\%$, we find

$$0.121 \text{ mb} \leq \sigma_1 \leq 0.181 \text{ mb},$$
$$0.332 \text{ mb} \leq \sigma_2 \leq 0.497 \text{ mb},$$
$$0.215 \text{ mb} \leq \sigma_3 \leq 0.321 \text{ mb},$$
$$0.0278 \text{ mb} \leq \sigma_4 \leq 0.0417 \text{ mb},$$
$$1.52 \text{ mb} \leq \sigma_5 \leq 2.28 \text{ mb}.$$

The experimental results show that $\sigma = 2 \pm 0.3$ mb,[1] so that it seems that a vector K' fits the experimental data better than a scalar K'.

Further evidence for this conclusion may be obtained from a re-examination of the work of Tiomno et al.[4] on the process $\pi^- + p \to K^0 + \Lambda^0$. We consider one K' exchange only, i.e., the diagram shown in Fig. 2, using the interactions (1) and (5) for the $K'K\pi$ vertex. With the incident pion kinetic energy 960 Mev, $m_0 = 878$ Mev, $\lambda = 23$ Mev, we get

$$\sigma = 0.0574\, g_S^2/4\pi,$$
$$\sigma = 0.00499\, g_{PS}^2/4\pi,$$

for scalar and pseudoscalar couplings, corresponding to K' with spin 0; and

$$\sigma = 1.24 G_V^2/4\pi,$$
$$\sigma = 0.871 G_{PV}^2/4\pi,$$

for vector and pseudovector couplings, corresponding to K' with spin 1. Taking the experimental total cross section as 0.67 mb $\leq \sigma \leq$ 0.93 mb,[5] we get

$$11.7 \leq g_S^2/4\pi \leq 16.2,$$
$$134 \leq g_{PS}^2/4\pi \leq 186,$$
$$0.54 \leq G_V^2/4\pi \leq 0.75,$$
$$0.79 \leq G_{PV}^2/4\pi \leq 1.07.$$

If we assume that $g_{K'YN} \leq g_{KYN} \leq g_{\pi NN}$, we see that either a vector K' or a pseudovector K' gives a good fit.

Curves for the differential cross section for the production of Λ^0 together with the experimental results of 83 events at the incident pion kinetic energy, 960 Mev,[5] are shown in Fig. 3. They are normalized to the same area under the experimental histogram by taking appropriate g's and G's. Again vector K' gives a good fit.

Though all the present calculations favor a

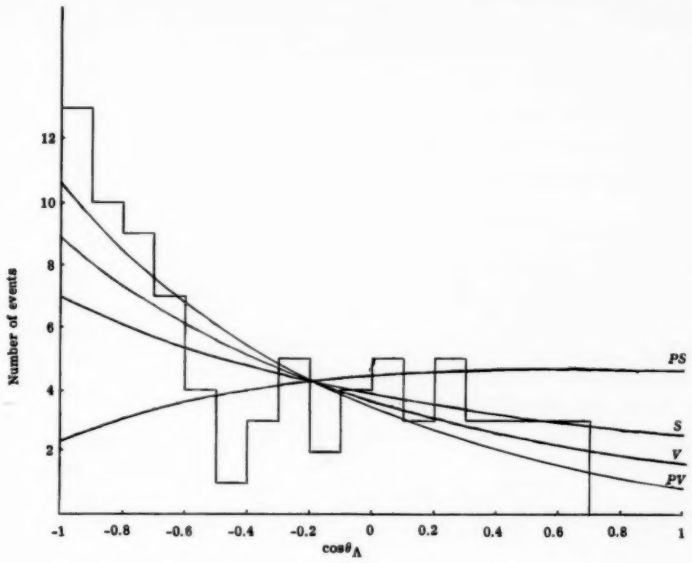

FIG. 3. Comparison of the experimental histogram of 83 events from bubble chamber, for the associated production $\pi^- + p \rightarrow K^0 + \Lambda^0$ at 960 Mev, with the theoretical predictions based on the assumption of exchange of a K'. All the curves are normalized to the same area under the histogram. Θ_Λ is the c.m. angle between the incident pion and Λ.

P-wave resonance, they do not exclude the possibility that K' can have spin zero because of the uncertainty of m_0 and λ.

The author wishes to express his gratitude to Professor A. Salam for suggesting this work and for many helpful discussions, and to Dr. P. T. Matthews for his interest.

[1]M. Alston et al., Phys. Rev. Letters 5, 520 (1960); also reported in the Proceedings of the Berkeley Conference on Strong Interactions, Berkeley, 1960 [Revs. Modern Phys. (to be published)].

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COUPLED INTEGRAL EQUATIONS OF THE OMNÉS-MUSKHELISHVILI TYPE

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(Received December 19, 1960)

In this note the Omnés method[1] of solution of certain singular integral equations is extended to the case of coupled equations. This problem arises in connection with many-channel γ reactions. The extension proposed here is the analog of Bjorken's generalization of the N/D method.[2]

Let us consider a many-channel reaction:

$$a + b \rightarrow c_1 + d_1$$
$$\cdots$$
$$c_n + d_n, \qquad (1)$$

and denote by f_i the matrix elements for these transitions.

Let us assume that the elastic channel is negligible as compared with the inelastic ones. The final-state interaction is thus restricted to transitions T_{ji} from a state $(c_i d_i)$ to a state $(c_j d_j)$. In this approximation unitarity gives in the physical region:

$$\mathrm{Im} f_i = \sum T_{ji}^* \rho_j f_j,$$

or in matrix notation:

$$\mathrm{Im} f = T^\dagger \rho f, \tag{2}$$

where ρ is the phase-space factor. Invariance of the interactions under time reversal implies that T_{ji} is symmetric.

Now assuming that the final-state interaction is known, we want to calculate the amplitudes f_i using the partial wave equations obtained from the Mandelstam representation. They are of the form,

$$f(\omega) = f_0(\omega) + \frac{1}{\pi}\int_{\omega_0}^{\infty} \mathrm{Im} f(\omega') \frac{d\omega'}{\omega' - \omega - i\epsilon}$$

$$= f_0(\omega) + F_+(\omega), \tag{3}$$

where the source function $f_0(\omega)$ is real in the physical region and assumed known. In general one can write $T(\omega) = N(\omega) D(\omega)^{-1}$, where $N(\omega)$ is real in the physical region and $D(\omega)$ is an analytic function of ω with a branch cut on the real axis in the interval (ω_0, ∞). From unitarity one obtains

$$\mathrm{Im} D = -\rho N \quad (\omega > \omega_0). \tag{4}$$

We then have

$$F_+ - F_- = 2i \,\mathrm{Im} f = 2iT^\dagger \rho f = 2iT^\dagger \rho (f_0 + F_+),$$

or

$$(1 - 2iT^\dagger \rho) F_+ - F_- = 2iT^\dagger \rho f_0.$$

Multiplying on the left by $D^\dagger$ and using (4), one obtains

$$D^T F_+ - D^\dagger F_- = 2iD^\dagger T^\dagger \rho f_0 = 2iN^T \rho f_0$$

$$= -2i \,\mathrm{Im} D^T f_0, \tag{5}$$

wherefrom one deduces

$$F_+(\omega) = -D(\omega)^{-1} T \frac{1}{\pi}\int_{\omega_0}^{\infty} \frac{d\omega'}{\omega' - \omega - i\epsilon} \mathrm{Im} D(\omega')^T f_0(\omega'). \tag{6}$$

Taking this result into (3), one obtains a solution for $f(\omega)$. If $f_0(\omega)$ is an analytic function of the form,

$$f_0(\omega) = \frac{1}{\pi}\int_\Gamma \frac{d\omega'}{\omega' - \omega} \alpha(\omega'),$$

then one can obtain directly a solution for $f(\omega)$ of the form,

$$f(\omega) = D(\omega)^{-1T} \frac{1}{\pi}\int_\Gamma D(\omega')^T \alpha(\omega') \frac{d\omega'}{\omega' - \omega}. \tag{7}$$

The question of uniqueness of (6) and (7) and of subtractions arises as for the single equation and can be discussed in much the same way.

The problem of determining $D(\omega)$ from the knowledge of the S matrix by means of the relation

$$S(\omega) = D(\omega)^* D(\omega)^{-1}, \tag{8}$$

was solved by Newton and Jost[3] for nonrelativistic potential scattering. In field theory $D(\omega)$ has the properties of the Jost functions, except, possibly, for the behavior at infinity. We shall show that one can extend the method of Newton and Jost to the case in which $D(\omega)$ does not approach 1 at infinity provided the S matrix satisfies certain conditions at $\omega \to \infty$.

Let $D_N(\omega)$ and $S_N(\omega)$ be the matrices obtained from $D(\omega)$ and $S(\omega)$ after the removal of the bound-state poles of $D(\omega)^{-1}$ is carried out, by the procedure of reference 3. Since $S_N(\omega)$ is still unitary and symmetric, one can write

$$S_N(\omega) = R(\omega) e^{2i\delta(\omega)} R(\omega)^{-1}, \tag{9}$$

where $R(\omega)$ is real and unitary and $\delta(\omega)$ (the eigenphase matrix) is diagonal. If the eigenphases do not vanish at infinity, then $S(\infty) \neq 1$ and $D(\infty) \neq 1$.

We shall introduce a matrix $\Delta(\omega)$ with all the relevant properties of $D_N(\omega)$, defined by

$$D_N(\omega) = R(\infty) E(\omega) \Delta(\omega) R(\infty)^{-1}, \tag{10}$$

where

$$E(\omega) = \exp\left(\frac{-1}{\pi}\int_{\omega_0}^{\infty} \frac{\omega - \omega_1}{\omega' - \omega_1} \frac{d\omega'}{\omega' - \omega - i\epsilon} \delta(\omega')\right). \tag{11}$$

For large ω, $E(\omega)$ behaves in the channel i like $\lambda_i (e^{-i\pi}\omega)^{\alpha_i}$, where λ_i is a real constant and $\pi\alpha_i = \lim_{\omega \to \infty} \delta_i(\omega)$. The existence of $R(\infty)$ is guaranteed by the properties of reality and unitarity.

Taking (10) into (8), one obtains

$$\Delta(\omega)^{*}\Delta(\omega)^{-1} = E(\omega)^{*-1}R(\infty)^{-1}S(\omega)R(\infty)E(\omega) = M(\omega). \quad (12)$$

Writing

$$R(\omega) = R(\infty)[1 + r(\omega)], \quad (13)$$

one can verify that $M(\infty) = 1$ provided the following condition is satisfied:

$$r_{ij}(\omega) = O(\omega^{-|\alpha_i - \alpha_j|}) \text{ for } \omega \to \infty. \quad (14)$$

Since it is true that

$$M(\omega)M(\omega)^{*} = 1, \quad (15)$$

the matrix $M(\omega)$ satisfies the conditions (5.4) of reference 3. If we assume that $\lim_{\omega \to \infty}\Delta(\omega)$ exists, then it will be a real matrix which can be chosen equal to 1. The function $\Delta(\omega)$ has then all the properties of the Jost functions. Therefore, one can apply the method of Newton and Jost to the determination of $\Delta(\omega)$ from $M(\omega)$.

The formalism described here applies to the problem of photodisintegration of the deuteron when the final proton-neutron system is in the triplet spin state and with a given total angular momentum. There are two channels corresponding, in the center-of-mass system, to parallel and antiparallel spin alignments.

For $J = 1$, $D(\omega)^{-1}$ has a pole corresponding to the deuteron bound state. The removal of this pole requires the knowledge of the asymptotic d- to s-wave ratio in the deuteron wave function or the ratio G/F of the invariant parameters of the npd vertex.[4]

Since the contribution of the inelastic channels has been neglected in this treatment, one cannot make use of the high-energy experimental data. The question of the behavior of $D(\omega)$ at infinity (which would depend on the behavior of the eigenphases) is then of purely theoretical interest.

I would like to thank Dr. H. M. Nussenzveig for a helpful discussion.

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ERRATUM

EXCHANGE POLARIZATION AND THE MAGNETIC INTERACTIONS OF RARE EARTH IONS. R. E. Watson and A. J. Freeman [Phys. Rev. Letters 6, 277 (1961)].

Figure 1 on page 278 is in error. The $(\rho_\uparrow - \rho_\downarrow)$ plotted there did not include the $5p$ spin density. The correct figure is shown below; note the change of scale at $r = 2.4$ a.u. to a common scale for both the $4f$ and the "core" spin densities.

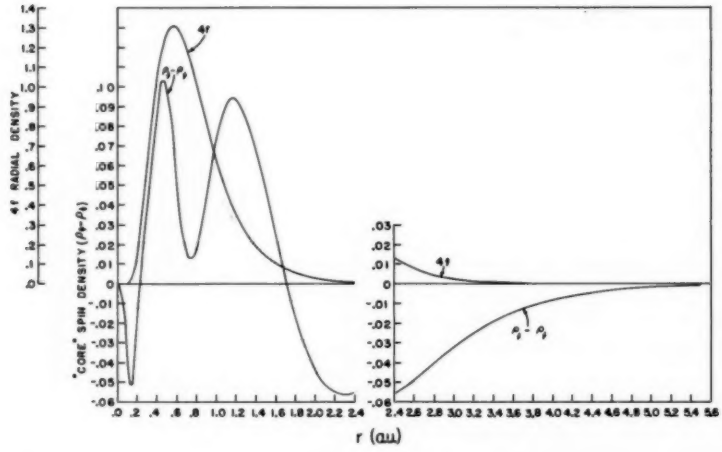

FIG. 1. The computed "core" electron spin density $(\rho_\uparrow - \rho_\downarrow)$ for all the electrons other than the $4f$ shell and, for comparison, the $4f$ density as well. Note the change of scales at $r = 2.4$ a.u. to a common scale for both $(\rho_\uparrow - \rho_\downarrow)$ and the $4f$ spin density.

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

STREAMER MECHANISM AND MAIN STROKE IN THE FILAMENTARY SPARK BREAKDOWN IN AIR AS REVEALED BY PHOTOMULTIPLIERS AND FAST OSCILLOSCOPIC TECHNIQUES. Gilbert G. Hudson* and Leonard B. Loeb, Department of Physics, University of California, Berkeley, California (Received June 1, 1960; revised manuscript received January 3, 1961).

A study has been made of the development of the luminosity in the transition from a corona or a Townsend predischarge to a filamentary spark in atmospheric air for a wide range of gap geometry extending from a positive needle point-to-plane gap to large sphere-to-plane gaps very close to the parallel plate case. Two photomultipliers viewed thin slices of the gap perpendicular to its axis: The one fixed near the anode triggered the Tektronix 517 oscilloscope and the other, which could be moved parallel to the gap axis, provided the signal for display. From the oscillograms, cross plots of the developing spatial distribution of luminosity across the gap were obtained for a number of gaps. It was found that, in general, a primary and a secondary dendrite, and even a tertiary, in some cases, develop out from the anode to the cathode at high speeds and prepare the way for the growth of the main stroke. In divergent fields the primary dendrite consists of a number of simultaneous streamer filaments which, in all cases, cross to the cathode, while the branch streamers of the secondary dendrite slow down in mid-gap and even fail to reach the cathode in the longer gaps. There are several sets of primary-secondary dendrites before each main stroke, in longer point-to-plane gaps, with about 200 μsec between sets, and about 1 μsec between the last set and the main stroke. While in the case of large sphere-to-plane gaps it was not possible to tell whether a dendrite consists of one or of several streamers, a primary (close to the cathode) and a secondary pulse can be observed in the longer gaps. With short gaps which are very close to the parallel plate case, only a secondary can be seen, and below about 1.0 to 1.3 cm a dendrite or streamer could not be resolved from the main stroke rise with the present equipment. In a few sphere-to-plane gaps oscillograms and cross plots were obtained which show the development of the main stroke as its toe crosses from the anode to within about 0.5 cm of the cathode, at which position the luminosity distribution of the fully developed main stroke shows a trough, with a maximum near the anode and a rise toward the cathode.

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STREAMER MECHANISM IN FILAMENTARY SPARK BREAKDOWN IN ARGON BY FAST PHOTOMULTIPLIER TECHNIQUES. L. B. Loeb, R. G. Westberg,* and H. C. Huang,† Department of Physics, University of California, Berkeley, California (Received June 1, 1960; revised manuscript received January 10, 1961).

Using techniques developed by Hudson on air and reported in the preceding paper, the phenomena were studied in Linde's spectroscopically pure grade Ar gas admitted to a system using Alpert vacuum techniques and on that gas further purified by gas cataphoresis. Study was made in a point-to-plane gap with a 2.36-mm hemispherically capped cylinder opposite a 3-cm distant thin outgassed Ni plane in the pressure range from 300 mm to 50 mm. For the spectroscopically pure grade Ar, transition from a positive point corona through a fine filamentary spark to an incipient arc breakdown on a time scale of 10^{-7} sec down to 100-mm pressure proceeds by movement of primary and secondary streamers progressing from anode to cathode at slower speeds and lower luminous intensities than air at 760 mm. Unlike air, the main stroke in Ar appears to move from anode towards the cathode. At 50 mm a somewhat diffuse spark channel did not reveal any streamer-like progression but the time scale was still in the 10^{-7}-sec range. One percent air in Ar at 60 mm restored streamers. Purified Ar at 240 mm revealed a 2-mm wide diffuse channel breakdown occurring across the whole gap by a process unknown with a rise time of several microseconds and sustained luminosity for tens of μsec with no indication of streamers. This demonstrates the necessity of adequate photoionizable impurities in Ar for the develop-

ment of the filamentary streamer spark transition.

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SPUTTERING THRESHOLDS. D. E. Harrison, Jr.,* and C. D. Magnuson, Physics Section, Convair, San Diego, California (Received January 27, 1961).

We attempt to give a logically coherent definition of the term "sputtering threshold," and establish criteria which may determine an experimental threshold. The Silsbee chain mechanism and the experimentally observed preferred direction of emission from single crystals are used to establish a threshold theory. Two models are required, one generally applicable when the mass ratio is less than one, and another when it is greater than one. Single-crystal threshold laws are obtained, and polycrystalline laws follow for face-centered cubic crystals by averaging the single-crystal forms. An approximate technique for the evaluation of surface atomic binding energies is presented so that the thresholds can be compared with experimental results. In all cases the theoretical thresholds are less than or comparable to experimental "thresholds."

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BEHAVIOR OF HOT ELECTRONS IN MICROWAVE FIELDS. B. V. Paranjape, Louisiana State University, Baton Rouge, Louisiana (Received December 23, 1960).

A strong external electric field in a semiconductor produces hot electrons. In the present paper we investigate theoretically the currents produced by such electrons in a microwave field. We discuss two special cases: Case A deals with a strong steady electric field on which a weak microwave field of frequency ω is superimposed. It is found that in addition to a steady current there is an alternating current of frequency ω which leads the microwave field by a phase given by Eq. (30). The phase difference is negligible at low frequency, but becomes appreciable at frequencies $\omega \approx 1/100\tau_0$, at liquid-nitrogen temperature. Here τ_0 is the electron-phonon relaxation time of thermal electrons. (Interaction with acoustic modes only is considered.) In Case B we consider the effect of a strong microwave field by itself. Here the current has a strong component of frequency ω and a weaker component of frequency 3ω. We find phases similar to those in Case A. Results for Case B are valid if $1/\tau_0 > \omega > 1/100\tau_0$ at liquid-nitrogen temperature.

ONE-DIMENSIONAL ORDER-DISORDER MODEL WHICH APPROACHES A SECOND-ORDER PHASE TRANSITION. George A. Baker, Jr., Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received January 30, 1961).

The calculation of the partition function for a simple one-dimensional order-disorder model is reduced to the solution of a certain functional equation. This equation is solved rigorously and it is shown that in the limit of indefinitely long-range interactions the model exhibits a finite discontinuity in the specific heat.

SPECTROSCOPIC STUDY OF THE YTTERBIUM-IRON EXCHANGE INTERACTION IN YTTERBIUM IRON GARNET. Kenneth A. Wickersheim, Hughes Research Laboratories, Malibu, California (Received January 26, 1961).

The exchange splittings of the ytterbium ground-state doublet and of an excited state ($J = 5/2$) doublet have been determined from spectroscopic studies of single-crystalline ytterbium iron garnet. The splittings were observed at 77°K as a function of the orientation of the magnetization of the crystal. The splittings exhibit the functional form of the g tensor of a two-level system. The principal values of the splittings (referred to local x, y, z axes) are, for the ground-state doublet, 11.6, 25.7, and 29.9 cm^{-1} and, for the excited state doublet, 15.9, 5.8, and 29.9 cm^{-1}.

From the ground-state splitting, using g values for ytterbium in yttrium gallium garnet, the approximate principal values of the exchange field (assumed to act on the ytterbium spin magnetic moment only) were found to be 349 000 gauss, 611 000 gauss, and 678 000 gauss. The principal values of the effective field (assumed to act on the total magnetic moment) are one fourth as

large as those of the exchange field.

On the basis of the ground-state exchange splitting, calculations of various macroscopic properties of ytterbium iron garnet, as a function of orientation of magnetization and of temperature, have been carried out by others. The calculated properties are in generally good agreement with experiment, but the sharp, low-temperature magnetic anisotropy anomalies which have been observed in ytterbium-doped yttrium iron garnet are not explained by the spectroscopic data.

NEUTRON DIFFRACTION INVESTIGATIONS OF METALLIC CERIUM AT LOW TEMPERATURES. M. K. Wilkinson, H. R. Child, C. J. McHargue, W. C. Koehler, and E. O. Wollan, Oak Ridge National Laboratory, Oak Ridge, Tennesse (Received January 16, 1961).

Neutron diffraction experiments have been performed on metallic cerium at a series of temperatures between room temperature and 4.2°K in an attempt to clarify the anomalous behavior which has been observed in previous specific heat and magnetic susceptibility measurements. Results on three specially prepared samples show that the interesting magnetic behavior can be correlated with the three crystallographic phases present in the samples. There is a change in the electronic configuration of the cerium atoms when the collapsed face-centered cubic phase is formed, and antiferromagnetic ordering occurs in the hexagonal close-packed phase at about 12.5°K.

MAGNETIC STRUCTURE OF CHROMIUM SELENIDE. L. M. Corliss, N. Elliott, J. M. Hastings, and R. L. Sass,* Department of Chemistry, Brookhaven National Laboratory, Upton, New York (Received January 31, 1961).

The magnetic structure of the NiAs-type compound, CrSe, has been determined by means of neutron diffraction. The indexing of superstructure lines which appear below the Néel point requires a unit cell three times as large as the conventional unit ($a = \sqrt{3}\, a_{\mathrm{NiAs}}$). Planes parallel to the basal plane contain three chromium atoms whose spins form an "umbrella"-like array with threefold symmetry. Individual moments alternate in sign along lines parallel to the c axis. A value of 2.90 μ_B has been deduced for the component of the chromium moment perpendicular to the c axis.

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ELECTRON MICROSCOPE OBSERVATION OF PRECIPITATES ON GROWN-IN DISLOCATIONS IN MgO. J. D. Venables, Parma Research Laboratory, Union Carbide Corporation, Parma, Ohio (Received January 30, 1961).

The defect structure of undeformed, single-crystal, Norton MgO has been examined by transmission in the electron microscope. It is observed that precipitate particles in the form of 0.2μ diameter spheres and short rods are present in large numbers on as-grown dislocations. Using special etching and optical techniques which are described, the concentration of the precipitate spheres is shown to average 10^9 "balls"/cc (but range from 10^7 to 10^{10} "balls"/cc) for a large number of samples examined. Qualitative evidence is presented which indicates that these precipitates are the origin of Tyndall scattering frequently observed in MgO.

MAGNETORESISTANCE OF ORIENTED GRAY TIN SINGLE CRYSTALS. O. N. Tufte* and A. W. Ewald, Department of Physics, Northwestern University, Evanston, Illinois (Received September 16, 1960).

The electronic band structure of gray tin was investigated through magnetoresistance measurements on oriented n- and p-type single crystals at 77, 195, and 273°K. From these measurements the low-field magnetoresistance coefficients were evaluated. Magnetoresistance anisotropy was observed in n-type crystals at 195 and 273°K but was not observed in either n- or p-type material at 77°K. Two possible explanations for the temperature-dependent anisotropy are proposed. The observed anisotropy satisfies, within the experimental uncertainty, the symmetry condition for ellipsoidal energy surfaces located along the [111] directions. Hall and conductivity measurements on samples used in the magnetoresistance study revealed a temperature-dependent mobility ratio greater than unity, which supports the assumption that the magnetoresistance anisotropy observed in the intrinsic range

should be assigned to the conduction band. Under this assumption and that of isotropy of lattice scattering, a lower limit of 2.3 is found for the electronic effective-mass anisotropy parameter.

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PARAMAGNETIC RESONANCE SPECTRA OF f^3 IONS IN A CUBIC SITE. G. Vincow and W. Low, Department of Physics, The Hebrew University, Jerusalem, Israel (Received January 23, 1961).

The paramagnetic resonance spectra of Nd^{3+} and U^{3+} in the cubic field of CaF_2 have been investigated at 3 cm at 20°K. In the case of Nd^{3+}, transitions within the lowest quartet $\Gamma_8^{(2)}$ and possibly in the next higher quartet $\Gamma_8^{(1)}$ have been observed. The angular behavior conforms with that predicted by Bleaney's formalism of the spin Hamiltonian of a Γ_8 state. In the case of U^{3+} there are considerable deviations of the experimental g values from the calculated ones. It is suggested that these deviations are caused by the stronger cubic field.

The efficiency of the thermal conversion from axial to cubic site is discussed. Additional lines suggest a new axial center along the [111] direction.

THERMAL CONDUCTIVITY OF CaF_2, MnF_2, CoF_2, AND ZnF_2 CRYSTALS. Glen A. Slack, General Electric Research Laboratory, Schenectady, New York (Received January 23, 1961).

The thermal conductivity of single crystals of CaF_2, MnF_2, CoF_2, and ZnF_2 has been measured over the temperature range from 3°K to 300°K. In this series CaF_2 and ZnF_2 are diamagnetic, whereas MnF_2 and CoF_2 are antiferromagnetic. All four crystals have nearly equal thermal conductivities at room temperature, but differ at lower temperatures. CaF_2, which is nearly isotopically pure, exhibits an exponential rise in conductivity with decreasing temperature characteristic of umklapp processes. ZnF_2 shows only traces of such umklapp behavior because its conductivity is limited by isotope and impurity scattering. Small cusps are observed in the conductivities of MnF_2 and CoF_2 at their Néel temperatures of 67°K and 38°K, respectively, which indicate the presence of phonon-magnon scattering. Some experimental details concerning thermal conductivity measurements and the behavior of gold-cobalt thermocouples are also given.

STRAIN-INDUCED EFFECTS ON THE DEGENERATE SPECTRAL LINE OF CHROMIUM IN MgO CRYSTALS. A. L. Schawlow, A. H. Piksis,* and S. Sugano,† Bell Telephone Laboratories, Murray Hill, New Jersey (Received January 26, 1961).

When uniaxial pressure is applied along [100], [110], and [111] directions to MgO crystals with chromium impurities, both a splitting and a shift of the purely cubic field fluorescence line at 14319 cm^{-1} are observed. The splitting is ascribed to the removal of the degeneracy associated with the $t_2^3\ ^2E$ excited state by the strain-induced low-symmetry crystal fields. A theoretical calculation of the splitting, assuming a point charge model, gives a surprisingly good agreement with the experiment. The shift is clearly due to the isotropic part of the strain-induced crystal fields, and a simple consideration shows that the observed red shift is caused by the strain-induced change of the Coulomb interaction between the t_2 electrons. Finally, limitations of the point-charge model adopted here are discussed.

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THEORY OF MAGNETOSTRICTION IN COBALT-MANGANESE FERRITE. John C. Slonczewski, International Business Machines Research Center, Yorktown Heights, New York (Received January 25, 1961).

The magnetostrictive effect of an orbitally degenerate magnetic ion in a cubic ferromagnet is calculated in detail using crystal-field theory. In contrast to our previous work it is not assumed that interatomic exchange energy is large compared with spin-orbit energy. The results are applied to the effect of cobalt in cobalt-manganese ferrite. It is found that the theory is consistent with experimental results for the magnetostrictive parameters λ_{100} and λ_{111} in the compound $Co_{0.245}Mn_{0.747}Fe_{1.99}O_4$ from 225 to 355°K. By fitting theory to experiment, a trigonal splitting of about 630 cm^{-1} for the ground state of the

cobalt ion is inferred. The trigonal splitting has less than half its value in cobalt-iron ferrite.

DISTRIBUTION OF THE MAGNETIZATION IN A FERROMAGNET. M. W. Muller, Central Research Laboratory, Varian Associates, Palo Alto, California (Received January 18, 1961).

The distribution of the magnetization is calculated for a thick ferromagnetic slab with easy axis transverse to the plane of the slab in a large applied field in the plane of the slab. The calculation predicts a stable nonuniform distribution which has several features suggestive of the domain pattern to be expected in such a system. In particular, the pattern consists of alternating strips having approximately the periodicity expected from conventional domain theory; and incipient flux closure domains appear if the anisotropy field is smaller than the demagnetizing field.

PHONON SCATTERING IN SODIUM CHLORIDE CONTAINING OXYGEN. Miles V. Klein,* Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York (Received January 30, 1961).

The thermal conductivity of supposedly pure NaCl crystals from several sources was found to vary by as much as two orders of magnitude at low temperatures. The conductivity of Harshaw crystals was particularly low. This effect was quantitatively related to the presence of an ultraviolet absorption band at 185 mμ known to be caused by oxygen-containing anionic impurities. Both phenomena were considerably reduced by treatment of the crystals in chlorine vapor at high temperatures; conversely both were enhanced by growing crystals from melts doped with NaOH, NaOD, and Na_2O_2. There was little evidence, however, that the dopants appeared in these forms in the crystals. Infrared measurements and pH titrations suggested that the most likely result of the dopings was to introduce carbonate into the crystals. The active impurity scattered phonons very strongly at low temperatures, at 5°K approximately 3000 times more strongly than is usually observed for point defects. The cross section was proportional to the first power of the phonon wave vector and was found to be independent of the defect concentration. No detailed model was found to explain these results. A likely explanation would be in terms of an interaction between the phonon field and localized modes of the scattering center.

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ULTRAVIOLET ABSORPTION OF THE MIXED SYSTEM KCl-KBr. Herbert Mahr, Laboratory of Atomic and Solid-State Physics, Cornell University, Ithaca, New York (Received January 27, 1961).

The ultraviolet absorption of thin evaporated layers and the edge absorption of single crystals of the mixed system KCl-KBr were measured at room temperature in the photon energy range from 6 to 11 ev. In the thin layers both the Cl^- and the Br^- absorption bands are present in the mixtures and their heights depend on the relative concentrations of the two ions. The energy values of the bands generally shift with composition although, in contrast to all the other bands, the high-energy KBr band shifts very little. The results obtained for the low-energy absorption bands can be described by a simple semiclassical electron transfer model. The behavior of the high-energy KBr band is discussed.

STUDY OF THE LOW-TEMPERATURE TRANSITION IN MAGNETITE AND THE INTERNAL FIELDS ACTING ON IRON NUCLEI IN SOME SPINEL FERRITES, USING MÖSSBAUER ABSORPTION. R. Bauminger, S. G. Cohen, A. Marinov, S. Ofer, and E. Segal, Department of Physics, The Hebrew University, Jerusalem, Israel (Received January 27, 1961).

A study has been made of the internal fields acting on Fe^{57} nuclei in some spinel ferrites, with particular reference to the low-temperature order-disorder transition in magnetite, using the techniques of Mössbauer absorption. For the Fe^{3+} ions at both the octahedral and tetrahedral sites in nickel ferrite ($NiFe_2O_4$) at 300°K, γFe_2O_3 at 85°K and 300°K, and magnetite (Fe_3O_4) at 85°K, the effective magnetic field at the Fe^{57} nuclei is the same and equal to about 5.1×10^5 oe. In magnetite, the value of H_{eff} in the Fe^{2+} ions is about 4.5×10^5 oe at 85°K. Measurements on Fe_3O_4 at room temperatures provide a microscopic confirmation of Verwey's hypothesis that

above the transition temperature of magnetite there is a fast exchange between the ferrous and ferric ions in the octahedral sites.

EFFECT OF TEMPERATURE AND DOPING ON THE REFLECTIVITY OF GERMANIUM IN THE FUNDAMENTAL ABSORPTION REGION. Manuel Cardona and H. S. Sommers, Jr., Laboratories RCA Limited, Zurich, Switzerland (Received January 6, 1961). The 2.1- and 4.4-ev peaks in the reflectivity spectrum of germanium have been studied as a function of temperature and doping. The temperature coefficient of the 2.1-ev peak is -4.2×10^{-4} ev/°K, that of the 4.4-ev peak -1.8×10^{-4} ev/°K. Large donor concentrations give a decrease in the energy gap at the zone boundary in the [111] direction which is nearly the same as the shrinkage of the thermal gap. No shift with doping of the 4.4-ev peak was detected. The spin-orbit splitting of the valence band at the L_3' point in the zone boundary was found to be 0.18 ev.

HYPERFINE STRUCTURE OF Fe^{57} IN YTTRIUM IRON GARNET FROM THE MÖSSBAUER EFFECT. C. Alff* and G. K. Wertheim, Bell Telephone Laboratories, Murray Hill, New Jersey (Received January 24, 1961).

The hyperfine structure of Fe^{57} in yttrium-iron garnet $[Y_3Fe_2(FeO_4)_3]$ has been obtained through the Mössbauer effect. A 0.002-in. thick slice of a single crystal of yttrium iron garnet, cut normal to a [110] direction, was used as an absorber of recoil-free gamma rays emitted by a stainless steel source. The iron in yttrium iron garnet is located in two nonequivalent sites, tetrahedral and octahedral, each of which has an axially symmetric electric field gradient. Data were taken with the magnetization aligned in a [111] and in a [100] direction in order to produce the simplest absorption pattern. For each direction of magnetization, the absorption lines of Fe^{57} at both sites have been resolved. The magnetic field at an Fe^{57} nucleus has been found to be 3.9×10^5 oe at a tetrahedral site and 4.7×10^5 oe at an octahedral site when the crystal is at room temperature (~300°K). The quadrupole coupling was found to be 7.5×10^{-8} ev in the tetrahedral site and 9.0×10^{-8} ev in the octahedral site.

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MECHANISM FOR PRODUCTION OF INTERSTITIALS IN KCl BY X RAYS AT LOW TEMPERATURES. R. E. Howard, National Bureau of Standards, Washington, D. C., Seymour Vosko, McMaster University, Hamilton, Ontario, Canada, and R. Smoluchowski, Princeton University, Princeton, New Jersey (Received January 30, 1961).

Experimental evidence indicates that halogen vacancies and interstitials may be formed by x irradiation of KCl at low temperatures. The validity of a mechanism based on multiple ionization depends upon several factors, among them efficiency of ionization and the availability of sufficient kinetic energy to remove the interstitial from the immediate vicinity of the vacancy. These two conditions are considered in some detail and found to be satisfied.

MAGNETIC SUSCEPTIBILITY OF CEROUS MAGNESIUM NITRATE. R. P. Hudson and W. R. Hosler, National Bureau of Standards, Washington, D. C. (Received January 30, 1961).

The most striking features of the behavior of cerous magnesium nitrate at liquid helium temperatures—extremely anisotropic susceptibility with $g_\parallel \approx 0$, a large temperature-independent term in $\chi_\perp$, the spin-lattice relaxation time varying as the twelfth or higher power of T in the region of 2°K—have until very recently received no detailed explanation. A measurement of $\chi_\perp$ between 4°K and 300°K was undertaken to elicit information on the energies of the excited doublets within the $J = 5/2$ ground multiplet, and thus to provide a guide for the reassessment of the crystal field parameters. We find δ_1 and δ_2 to be 30 and (roughly) 200 cm^{-1}, respectively, in contrast to the 113 and 150 cm^{-1} of existing theory. The results are at variance with the published data for $\chi_\perp$ above 85°K, but are in fairly good agreement with the recent findings of Leask and Wolf at low temperatures.

MOLECULAR MODEL OF THE HEISENBERG EXCHANGE INTERACTION. R. K. Nesbet, Department of Physics, Boston University, Boston, Massachusetts (Received January 25, 1961).

The electronic wave function of N_2 has been calculated for a series of internuclear distances,

in the simplest LCAO approximation, including the principal effects of configuration interaction. As the internuclear distance increases, there is a well-defined sequence of regions in which the ground state is most closely approximated by configurations in which successively more orbitals are represented as localized functions (definitely associated with one of the two atoms) rather than as the odd or even linear combinations of these appropriate to the full molecular symmetry. This corresponds to a continuous change in the nature of the unrestricted Hartree-Fock valence orbitals from molecular to atomic character as the atoms are separated. In the intermediate range of internuclear separation, it is a better approximation to treat some of the valence orbitals as modified atomic orbitals, coupled by an antiferromagnetic Heisenberg exchange interaction, than as molecular orbitals. Various contributions to the Heisenberg "exchange integral," of the kinds considered for the transition metals, are evaluated and compared. It is found that the ordinary direct exchange (which leads to ferromagnetic coupling) is small compared with the sum of the various antiferromagnetic effects, none of which can be described within the traditional Hartree-Fock approximation, which for solids becomes the energy band theory. A method is proposed by which the magnetic interaction in solids could be evaluated quantitatively, by modifying the usual energy band calculations in the same way that the usual molecular orbital theory is modified in the present work. Similar refinements to the band theory of the transition metals have recently been proposed by Goodenough on empirical grounds.

MICROWAVE ZEEMAN SPECTRUM OF ATOMIC FLUORINE. H. E. Radford,* V. W. Hughes, and V. Beltran-Lopez, Gibbs Physics Laboratory, Yale University, New Haven, Connecticut (Received January 19, 1961).

A paramagnetic resonance absorption spectrum has been observed in the products of a radio-frequency electrodeless discharge in fluorine gas. The spectrum consists of eight lines, widely spaced over a magnetic field range 2000 - 6000 gauss, which can be identified with $\Delta M_F = \pm 1$ transitions in the ground $^2P_{3/2}$ level of the F^{19} atom. The analysis of the spectrum yields $-g_J(F;\ ^2P_{3/2})/g_p = 438.4839 \pm 0.0003$, $\Delta\nu(F;\ ^2P_{3/2}) = 4020.01 \pm 0.02$ Mc/sec, and $-a'''(F;\ ^2P)/h = 446 \pm 10$ Mc/sec, where a''' is the coupling constant of the off-diagonal hyperfine interaction in the ^2P term and g_p is the g factor of protons in a cylindrical sample of mineral oil.

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FLUORESCENT RESPONSE OF SCINTILLATION CRYSTALS TO HEAVY IONS. A. M. Smith, E. Newman, and F. E. Steigert, Yale University, New Haven, Connecticut (Received January 27, 1961).

The light output of CsI(T1) was measured as a function of energy for incident ions of B^{10}, B^{11}, C^{12}, N^{14}, O^{16}, and F^{19}. The response of NE 102 plastic and anthracene scintillators was also measured for ions of N^{14} and O^{16}, respectively. The response of CsI was essentially linear for energies above 6 Mev/A, where A is the mass number of the incident ion. The NE 102 was linear for energies above 4 Mev/A. The anthracene data showed slight curvature even at 9 Mev/A. It would appear that the response of CsI differs somewhat among crystal samples.

INTRINSIC EXCITED STATES IN Hf^{178} POPULATED BY THE ALLOWED DECAY OF 9.3-min Ta^{178}. C. J. Gallagher, Jr., H. L. Nielsen, and O. B. Nielsen, Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark (Received January 26, 1961).

The levels in Hf^{178} populated by the allowed positron and electron capture decay of 9.3-min Ta^{178} have been investigated with the Copenhagen six-gap spectrometer. Conversion-electron spectrum and beta-gamma coincidence measurements have established the spins and parities of levels in Hf^{178} with energies (in kev) of 93 (2+), 307 (4+), 1197 (0+), 1277 ((2+)), 1430 ((1+)), 1440 (0+), and 1483 (2+), and possibly a level at 1550. (Double parentheses indicate that the spin is not definitely established.) Intrinsic configuration assignments for the excited states are discussed in terms of recent theoretical developments. The half-life of the 93-kev level has been measured and found to be $(1.25 \pm 0.08) \times 10^{-9}$ sec. The β^+/K capture ratio measured for the allowed decay to ground is within experimental error of theory. Reduced $E0/E2$ transition probability ratios for the transitions depopulating the 1197- and 1440-kev 0+

levels have been calculated from the observed K-conversion line intensities of the $E0$ and $E2$ transitions depopulating the levels.

COHERENT SCATTERING OF 1.17-Mev and 1.33-Mev GAMMA RAYS THROUGH SMALL ANGLES. P. P. Kane* and G. M. Holzwarth,† Scott Laboratory, Wesleyan University, Middletown, Connecticut (Received April 29, 1960; revised manuscript received January 3, 1961).

The dependence of the differential cross section for the coherent scattering of 1.17-Mev and 1.33-Mev gamma rays on atomic number was investigated. An empirical procedure, which makes an absolute determination of the cross sections unnecessary, was used to estimate the Compton scattering cross sections. The latter were subtracted from the measured cross sections in order to obtain the relative coherent scattering cross sections, which were found to vary as Z^n. Our average value for n is 3.07 ± 0.18

The angular distribution of the total (coherent and Compton) scattering cross section was also investigated in the case of copper and lead between 2.43° and 5.79°. Our results are compared with the existing theoretical predictions and with the results of the earlier experiments, wherever the latter are available.

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DECAY OF I^{134}. Noah R. Johnson, E. Eichler, G. D. O'Kelley, J. W. Chase, and J. T. Wasson,* Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received January 30, 1961).

The decay properties of 53-minute I^{134} have been investigated with scintillation techniques as part of a program for the systematic study of xenon energy levels. Energies (and intensities) of the gamma rays determined from the single-crystal and coincidence studies are 0.135 (3.2), 0.18, 0.23, 0.27, 0.32, 0.39 (7.2), 0.41 (0.6), 0.43 (2.9), 0.51 (0.9), 0.54 (8.4), 0.61 (19), 0.69 (7.3), 0.75 (1.3), 0.77 (6.0), 0.848 (100), 0.864 (4.6), 0.890 (74), 0.96 (2.0), 1.00 (4.7), 1.07 (18), 1.15 (10), 1.28 (1.4), 1.34 (1.5), 1.46 (3.7), 1.49 (1.0), 1.62 (4.9), and 1.79 (4.9) Mev. There are two gamma rays at each energy of 0.89 and 1.07 Mev. The single-crystal spectra were corrected experimentally for gamma-ray summing. Gamma coincidence spectra were measured by gating at energies of 0.135, 0.41, 0.61, 0.85, 0.89, 1.00, 1.07, 1.15, 1.46, 1.62, and 1.79 Mev in the gamma-ray spectrum. Beta-ray spectra were measured in coincidence with gamma rays at 0.85, 1.00, 1.07, 1.15, 1.46, 1.62, and 1.79 Mev. These measurements and the single-crystal data disclosed beta rays with end-point energies of 2.41, 2.21, 1.68, 1.49, 1.25, and 1.05 Mev. In a three-crystal "beta-gamma-gamma" experiment the 2.41-Mev beta-ray group was shown to populate a level in Xe^{134} at 1.74 Mev; therefore, the energy difference between the ground states of I^{134} and Xe^{134} is 4.15 ± 0.06 Mev.

A decay scheme is proposed with energy levels (and spins) in Xe^{134} at 0.85 (2+), 1.62 (2+), 1.74 (4+), 1.92, 2.34, 2.43, 2.48, 2.64, 2.88, 3.11, 3.30, and 3.41 Mev. A collective nature of the low-lying levels is suggested in that the 1.62- and 1.74-Mev states appear to be members of a "vibrational" doublet at about twice the energy of the first excited state.

The half-life of I^{134} was redetermined as 52.8 ± 0.3 minutes.

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NUCLEAR SPECTROSCOPY OF Ta^{181}. Arthur H. Muir, Jr.,* and F. Boehm, California Institute of Technology, Pasadena, California (Received January 16, 1961).

The nuclear levels of Ta^{181} were investigated by a study of the β decay of Hf^{181} and the electron capture decay of W^{181}. Evidence for weak M-shell conversion lines of an ~6-kev transition in the Hf^{181} was found with a β spectrometer. An investigation of the W^{181} decay with this instrument revealed strong M-shell conversion lines corresponding to a 6.25 ± 0.3 kev transition. With the aid of additional evidence, the conclusion is made that the 476-kev transition in the Hf^{181} decay occurs between the 482-kev level and a new level at 6 kev. Using an argon proportional counter, a 6-kev γ ray was also found in the W^{181} decay. The conversion coefficient of this transition was determined to be $\alpha_T = 44 \pm 7$. This conversion coefficient and the M-subshell conversion ratios indicate that the 6-kev transition is of $E1$ multipolarity. The 6-kev level is as-

signed as the 9/2- [514] Nilsson intrinsic state. It is also concluded that the previously reported 152-kev transition in the W^{181} decay occurs between a new 11/2- ($K = 9/2$-) rotational level at 158 kev and the 6-kev level. From a measurement of the tantalum L/K x-ray intensity ratio, the W^{181} decay energy is found to be 176 $^{+44}_{-22}$ kev. The branchings of this decay to the various Ta^{181} levels are as follows: 158 kev (0.11%), 136 kev (0.067%), 6.25 kev (~35%), and ground state (~65%). All findings and proposals are consistent with predictions of the unified model of the nucleus.

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$Ce^{140}(d, p)Ce^{141}$ REACTION. G. B. Holm* and H. J. Martin, Jr.,† Indiana University, Bloomington, Indiana (Received January 30, 1961).

Eight proton groups were observed, leading to levels in Ce^{141} at excitation energies of 0, 0.65, 1.12, 1.35, 1.47, 1.77, 2.15, and 2.41 Mev. Angular distributions and relative cross sections were used to make the following assignments: $7/2^-$ for the ground state; $3/2^-$ for the 0.65-Mev state; $3/2^-$ for the 1.12-Mev state; $5/2^-$ for the 1.77-Mev state; and $1/2^-$ for the 2.41-Mev state.

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POSSIBLE PARITY AND TIME-REVERSAL EXPERIMENTS USING THE MÖSSBAUER EFFECT. M. Morita, Department of Physics, Columbia University, New York, New York (Received January 30, 1961).

In the successive transition of the beta and gamma decays, the excited and ground states of the daughter nucleus are effectively polarized, when the satellites of the Mössbauer effect are separately observed. Using this nuclear polarization, we design various experiments to detect parity nonconservation and time-reversal invariance in beta decay. These experiments involve the measurement of the coincidence counting rate of beta rays and satellites of the Mössbauer effect. The resulting improvement in accuracy will make possible, e.g., the precision measurement of the asymmetry of beta-ray angular distributions.

NUCLEAR MOMENTS AND ISOTOPE SHIFTS OF Tl^{199}, Tl^{200}, Tl^{201}, Tl^{202}, AND Tl^{204}—ISOTOPE SHIFTS IN ODD-ODD NUCLEI. R. J. Hull and H. H. Stroke, Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received January 30, 1961).

The hyperfine-structure separations and isotope shifts of several radioactive isotopes of thallium have been measured by optical spectroscopic techniques. The results are: $\mu^{199} = 1.57$ nm; $\mu^{201} = 1.58$ nm; and both $|\mu^{200}|$ and $|\mu^{202}| \leq 0.15$ nm. The isotope shift measurements, which include the first data of this kind obtained for heavy odd-odd nuclei, permitted a comparison of the relative isotope shifts for isotones in mercury and thallium. A marked similarity in the shifts was observed.

MECHANISM OF FISSION OF HEAVY NUCLEI. Peter Fong, Physics Department, Utica College of Syracuse University, Utica, New York (Received October 31, 1960).

Question is raised concerning the validity of the Vladimirskii mechanism of fission in which the individual nucleons with large component of angular momentum in the direction of the symmetry axis give rise to instability against asymmetric deformation and thus lead to an asymmetric saddle point.

STATISTICAL THEORY OF NUCLEAR FISSION AND PROMPT NEUTRON DISTRIBUTION. Peter Fong, Department of Physics, Utica College of Syracuse University, Utica, New York (Received October 31, 1960; revised manuscript received March 6, 1961).

It is shown that the statistical theory of nuclear fission is consistent with the recent experimental results on prompt neutron distribution in fission if we assume the existence of some constraint in the process of approaching equilibrium which controls the partition of excitation between the two fragments.

NUCLEAR MODELS AND NUCLEAR FISSION. Peter Fong, Physics Department, Utica College of Syracuse University, Utica, New York (Received October 31, 1960).

The hindrance to spontaneous fission by the odd nucleon in the fissioning nucleus may be explained as due to the pairing energy of the odd nucleon at the saddle point deformation.

NEUTRON GROUPS FROM $K(\alpha,n)Sc$. A. M. Smith and F. E. Steigert, Yale University, New Haven, Connecticut (Received January 27, 1961).

Neutron groups resulting from the alpha-particle bombardment of separated isotopes of potassium have been observed. Ground-state Q values of -3.42 ± 0.06 Mev for $K^{41}(\alpha,n)Sc^{44}$ and -7.16 ± 0.06 Mev for $K^{39}(\alpha,n)Sc^{42}$ were obtained. A large number of excited states or groupings of states were also observed. The presence of chlorine in one of the targets permitted measurement of the $Cl^{37}(\alpha,n)K^{40}$ reaction as well. A ground-state Q value of -3.86 ± 0.06 Mev was obtained. A tentative value of -5.89 ± 0.06 Mev can be given for the $Cl^{35}(\alpha,n)K^{38}$ ground-state Q value.

GAMMA RADIATION FROM LOW LEVELS OF Al^{27}. R. D. Bent and W. W. Eidson, Indiana University, Bloomington, Indiana (Received January 16, 1961).

The $Al^{27}(\alpha,\alpha'\gamma)$ reaction was investigated using particle-gamma coincidence techniques and the 22-Mev alpha-particle beam from the Indiana University cyclotron. A 0.79 ± 0.03 Mev gamma-ray transition between the 3.0- and 2.21-Mev states of Al^{27} was observed. This result, together with other data, suggests that the 3.0- and 2.21-Mev states are the $9/2^+$ and $7/2^+$ members of a $K=5/2$ rotational band.

ANGULAR DISTRIBUTIONS OF $T(p,n)He^3$ NEUTRONS FOR 3.4- TO 12.4-Mev PROTONS. M. D. Goldberg, J. D. Anderson, J. P. Stoering, and C. Wong, Lawrence Radiation Laboratory, University of California, Livermore, California (Received January 30, 1961).

Angular distributions of the neutrons from the $T(p,n)He^3$ reaction have been obtained for incident laboratory proton energies of 3.4, 4.3, 5.0, 6.5, 8.0, 8.8, 10.3, 11.5, and 12.4 Mev. The neutrons were detected by a plastic scintillator, and standard time-of-flight techniques were used to separate the monoenergetic neutron group from background neutrons and gamma rays. The distributions in the center-of-mass system show substantial backward peaking. Above about 8-Mev proton energy, a broad maximum appears at about 80° (c.m) and persists through the highest energy measured. The absolute 0° cross sections and the total integrated cross sections are in excellent agreement with previous measurements.

BETA DECAY OF NATURALLY RADIOACTIVE In^{115}. G. B. Beard, Department of Physics, Wayne State University, Detroit, Michigan, and W. H. Kelly, Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan (Received January 23, 1961).

A liquid scintillator loaded with indium has been used to study the fourth forbidden beta decay of In^{115}. Specific activity measurements yield a half-life of $(6.9\pm1.5)\times10^{14}$ years. A crude beta spectrum was obtained. Linear extrapolation of the Fermi-Kurie plot gives an end-point energy of 625 ± 70 kev.

COUPLED SQUARE WELL MODEL FOR ELASTIC SCATTERING. D. E. Bilhorn, Rice University, Houston, Texas, and W. Tobocman,* Case Institute of Technology, Cleveland, Ohio (Received January 30, 1961).

A simple model for s-wave neutron scattering is provided by representing the scattering potential by a pair of coupled square wells. Such a model produces resonances that exhibit the giant resonance effect. We have compared isolated resonances given by this model for two types of coupling with the Breit-Wigner formula. We find that for a resonance with a width of about 16 kev the resonant part of the scattering does indeed have the Breit-Wigner form. The resonance energy is found to be considerably shifted from the energy of the bound state that exists in the zero-coupling-strength limit. Also, the nonresonant part of the scattering amplitude is considerably different from both the hard-sphere scattering amplitude and the zero-coupling-strength limit scattering amplitude. This last result is in

accord with expectations based on R-matrix theory.

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INVESTIGATIONS OF THE REACTION $Cl^{35}(n,\gamma\gamma')Cl^{36}$. James E. Draper and Allan A. Fleischer,* Yale University, New Haven, Connecticut (Received January 23, 1961).

Two-step gamma-ray cascades to the ground state of Cl^{36} following thermal neutron capture by Cl^{35} have been investigated with the sum-coincidence apparatus. The direct experimental result is the product $I_1 b_2$ of the intensity I_1 of the initial transition and the branching factor b_2 of the intermediate state to the ground state. The quantity b_2 is separately deduced from auxiliary information about I_1. The lower energy members of the stronger cascades occur at 0.79, 1.16, 1.60, 1.96, and 2.87 Mev with values of $I_1 b_2$, respectively, of 8.2, 11.6, 2.9, 12.1, and 5.1 per 100 neutrons captured. Weaker cascades appear at 2.2, 2.48, 2.6, 2.68, and 3.05 Mev. Cascades appearing between 3.3 and 4.3 Mev have $I_1 b_2 \leq 0.5\%$. The b_2 following the strongest of all initial transitions, viz., 6.11 Mev, is only ≤ 0.02. Approximately 46% of all neutrons captured produce two-step cascades in Cl^{36}.

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NEUTRON-DEFICIENT NUCLIDES OF HAFNIUM AND LUTETIUM. Erich R. Merz* and Albert A. Caretto, Jr., Department of Chemistry, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received January 23, 1961).

New neutron-deficient nuclides of lutetium and hafnium were produced by bombarding lutetium oxide with 300- to 400-Mev protons. The genetic relationships and mass assignments were established by means of high-purity chemical separations and a series of chemical isolation experiments in which the daughter activity was determined as a function of time.

The positron spectra of the different nuclides were measured with an anthracene crystal detector and 256-channel pulse-height analyzer. Gamma radiation was also observed for Lu^{168}, Lu^{169}, Lu^{170}, Hf^{168}, and Hf^{169} by means of a NaI-crystal detector and the pulse-height analyzer. The half-lives and maximum positron radiation observed are: Lu^{168}, $T_{1/2} = 7.0$ min, $E_{\beta+} = (1.20 \pm 0.05)$ Mev; Lu^{169}, $T_{1/2} = 1.5$ days; Lu^{170}, $T_{1/2} = 1.9$ days; $E_{\beta+} = (1.8 \pm 0.1)$ Mev; Hf^{168}, $T_{1/2} = 22$ min; $E_{\beta+} = (1.7 \pm 0.1)$ Mev; Hf^{169}, $T_{1/2} = 1.5$ hours; Hf^{170}, $T_{1/2} = 9$ hours.

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NONEXISTENCE OF A 9.0-Mev LEVEL IN C^{12}. D. E. Alburger and D. H. Wilkinson,* Brookhaven National Laboratory, Upton, New York (Received January 16, 1961).

An energy level in C^{12} at 9.0 Mev has been reported as a result of (p,γ,γ) triple coincidence measurements on the $B^{10}(He^3,p)C^{12}$ reaction at $E_{He^3} = 2.2$ Mev. This reaction has been reinvestigated in a similar experimental arrangement by using alternately Pilot-B, CsI, and NaI scintillators for detection of the protons. Only the Pilot-B, which had been used in the previous work, exhibits the proton group corresponding to a "9.0-Mev level" in C^{12}. The triple coincidence effect in this case is actually due to the intense ~17-Mev protons in the $B^{10}(He^3,p)C^{12}$ reaction leading to the 4.43-Mev first excited state of C^{12} which, upon entering the scintillator, can inelastically scatter from carbon and produce secondary gamma radiation of 4.43 Mev. The net energy deposited in the scintillator has the appearance of a proton group to a 9.0-Mev level in C^{12} in triple coincidence with two 4.43-Mev gamma rays. The magnitude of the effect is calculated from published cross sections for inelastic scattering and it agrees with the apparent population intensity of the nonexistent "9.0-Mev level."

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CORE EXCITATIONS IN NONDEFORMED, ODD-A NUCLEI. A. de-Shalit,* Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received January 16, 1961).

The possibility of describing some excited states of odd-A nuclei in terms of excitations of the even-even core is investigated. No assumption is made on the nature of the core ex-

citation, but certain relations involving electromagnetic transitions and moments are deduced. These seem to fit well some data available on Ag^{107}, Ag^{109}, Au^{197}, Hg^{199}, Tl^{203}, and Tl^{205}. More experimental data are required to test the validity of this picture in other cases.

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PHOTODISSOCIATION OF THE μ MESON. M. E. Ebel and W. D. Walker, University of Wisconsin, Madison, Wisconsin (Received May 25, 1960; revised manuscript received March 6, 1961).

The cross section for the production of a charged vector boson by the dissociation of a high-energy μ-meson beam undergoing Coulomb scattering has been calculated. The cross section for such a process is $(1\text{-}3)\times10^{-34}$ cm^2 in Pb. Possible experiments for the detection of such decays are discussed.

HYPERON DECAY IN THE NONLEPTONIC MODE. Seitaro Nakamura, Department of Physics, University of Tokyo, Tokyo, Japan and Nihon University, Tokyo, Japan, and Michiji Konuma, Department of Physics, University of Tokyo, Tokyo, Japan (Received September 27, 1960; revised manuscript received November 30, 1960).

An interaction Hamiltonian of the weak interaction between baryons and pions is described in the three-dimensional charge space. In order to determine the interaction types, a conservation law of extended chirality is speculated; the branching ratios and the absolute lifetimes of the hyperon decay in the nonleptonic modes are calculated and the results are compared with experiment.

PHOTOPRODUCTION OF MESONS FROM HYDROGEN AND DEUTERIUM. Robert W. Kenney, Edward A. Knapp,* Victor Perez-Mendez,† and Walton A. Perkins,‡ Lawrence Radiation Laboratory, University of California, Berkeley, California (Received January 26, 1961).

The relative yields of positive pions produced from hydrogen and deuterium by the 340-Mev bremsstrahlung beam of the Berkeley synchrotron have been measured in the laboratory system at angles of 20, 40, and 60 deg, and at pion energies ranging from 45 to 145 Mev. The ratio of the relative yields of pions from deuterium and hydrogen was roughly constant as a function of angle, but decreased monotonically with pion energy from a value of 0.90 ± 0.05 at 45 Mev to a value of 0.55 ± 0.07 at 145 Mev.

Comparison with the phenomenological theory of Chew and Lewis indicates a gradual change from nucleon spin flip near threshold to no-spin-flip transitions above 140 Mev. Comparison with Uretsky's calculation involving final states shows fair agreement with plane-wave and shape-independent approximations. Poor agreement with the zero-range approximation shows that final-state interactions are important in the theory of photoproduction of pions from deuterium.

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PHASE-PARAMETER REPRESENTATION OF NEUTRON-PROTON SCATTERING FROM 13.7 TO 350 Mev. M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, Yale University, New Haven, Connecticut (Received January 30, 1961).

Results of gradient searches, by means of an IBM-704 machine, for phase parameters representing neutron-proton scattering are reported. The analysis made use of most available measurements. The number of "measurements" used in the final searches was 293, with 35 additional ones which were used to obtain composite values. Most of the fits join reasonably smoothly to the $^3S_1$ phase-shift curve at low energies. The validity of charge independence was assumed, and the more probable among the $T=1$ phase-parameter sets obtained in a previously described series of searches for phase-parameter fits to p-p data were therefore employed for this T. The one-pion exchange values were used for the larger L and J. Independent sets of searches started with phase parameters corresponding to the Gammel-Christian-Thaler potential and the Gammel-Thaler potential, respectively. The procedure was varied by employing a weighted mean of fits obtained from the two different starting points as a new

starting set and other devices described in the text. The final fits are appreciably better than the starts, the mean square deviation being reduced by a factor ~20 in some cases. A rough division of the final fits into related families according to the behavior of the parameters K_3 and ρ_3 can be made. The value of additional measurements and especially those of the triple-scattering parameters and polarization correlation is pointed out. Tests on reasonableness of one of the better fits from the point of view of representation by a static potential have been made, with satisfactory results.

PHOTOPROTONS PRODUCED BY 245±15 Mev GAMMA RAYS ON CARBON. Robert J. Cence and Burton J. Moyer, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received January 23, 1961).

A carbon target was bombarded by 342-Mev bremsstrahlung from the Berkeley synchrotron. The energy spectrum of protons produced at 60 deg was measured from 90 Mev to 250 Mev. Interactions due to 245±15 Mev gamma rays were selected out by requiring a coincidence between the protons from the target and the electrons associated with these gamma rays. Analysis was made using the quasi-deuteron model of Levinger. In contrast with previous analyses of this kind, conservation of both momentum and energy were taken into account in a fairly accurate way. Results of this analysis determine the momentum distribution of the centers of mass of the quasi-deuterons. The momentum distribution that results in the observed proton spectrum, normalized to one quasi-deuteron, is given by

$$\frac{d^3N}{dP_D^3} = \frac{0.2}{(4\pi ME_1)^{3/2}} \exp(-P_D^2/4ME_1) + \frac{0.8}{(4\pi ME_2)^{3/2}} \exp(-P_D^2/4ME_2),$$

where $E_1 = 1.6$ Mev, $E_2 = 20$ Mev.

SINGLE SCATTERING OF 2-Bev/c MUONS IN NUCLEAR EMULSIONS. C. Y. Kim, S. Kaneko,* Y. B. Kim, G. E. Masek, and R. W. Williams, Department of Physics, University of Washington, Seattle, Washington (Received January 30, 1961).

The single scattering of high-energy muons from emulsion nuclei was measured using a monoenergetic beam of muons produced at the Berkeley Bevatron. The median momentum of muons was 2.00±0.03 Bev/c with a spread of no more than ±3.5%. Nuclear emulsion stacks were exposed to this muon beam behind the targets used for a counter experiment. The results of the counter experiment were reported earlier. The muon tracks recorded in nuclear emulsions were followed by a special fast-scanning technique, and a total of 682 single scattering events were found from 743 meters of track following. For the muon beam accepted in the emulsion scanning, the pion contamination was measured to be (1.3±0.2)%. These pions contribute to the integral muon scattering data by about 3% for scattering angles greater than 1°. The observed scattering distribution which extends up to 3° scattering angle, or momentum transfer of about 100 Mev/c, is in good agreement with the electromagnetic theory predictions and complements the counter experiment which covered momentum transfers of 70-400 Mev/c.

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DIRECT NUCLEON-NUCLEON COLLISIONS INSIDE THE NUCLEUS ACCORDING TO THE IMPULSE APPROXIMATION. Lester Winsberg and Thomas P. Clements, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received November 18, 1960).

Direct nucleon-nucleon collisions play an important role in high-energy nuclear reactions. The importance of such collisions at lower energies is not clear. To aid in the interpretation of nuclear reactions, we have analyzed the collisions between an incident nucleon and nucleons in a Fermi gas by means of the impulse approximation. The treatment given here is based on information from nucleon-nucleon scattering experiments. Collisions inside a nucleus are considered to be the same as those in the unbound state at the same center-of-mass energy, except for the effect of the Pauli exclusion principle. The effective elastic and inelastic cross section, $\langle\sigma\rangle$, between like and unlike nucleons is computed for incident energies from 10 Mev to 6 Bev at several values of the Fermi energy. The properties of the struck nucleons in allowed collisions are also calculated. This information

may prove useful in interpreting some recoil experiments. Analytical expressions for $\langle\sigma\rangle$ and quantities related to the struck nucleon are given for elastic collisions in which the scattering is isotropic and the free-particle cross sections either are constant or vary inversely as the bombarding energy.

TIME IN THE QUANTUM THEORY AND THE UNCERTAINTY RELATION FOR TIME AND ENERGY. Y. Aharonov and D. Bohm, H. H. Wills Physics Laboratory, Bristol, England (Received September 7, 1960).

Because time does not appear in Schrödinger's equation as an operator but only as a parameter, the time-energy uncertainty relation must be formulated in a special way. This problem has in fact been studied by many authors and we give a summary of their treatments. We then criticize the main conclusion of these treatments; viz., that in a measurement of energy carried out in a time interval, Δt, there must be a minimum uncertainty in the transfer of energy of the observed system, given by $\Delta(E' - E) \geq h/\Delta t$. We show that this conclusion is erroneous in two respects. First, it is not consistent with the general principles of the quantum theory, which require that all uncertainty relations be expressible in terms of the mathematical formalism, i.e., by means of operators, wave functions, etc. Secondly, the examples of measurement processes that were used to derive the above uncertainty relation are not general enough. We then develop a systematic presentation of our own point of view, with regard to the role of time in the quantum theory, and give a concrete example of a measurement process not satisfying the above uncertainty relation. Finally, we summarize the argument and give our general conclusions.

CUSP PHENOMENA IN THE REGION OF TWO NEIGHBORING THRESHOLDS. J. Sucher, G. A. Snow, and T. B. Day, Physics Department, University of Maryland, College Park, Maryland (Received January 23, 1961).

Previous discussions of the cusp phenomena at the threshold for a new reaction are extended to the case of two neighboring thresholds. The S matrix is constructed from an $n \times n$ κ matrix in such a way as to insure that the physical S matrix is unitary when only r of the n channels are open. As a special application, the amplitude for the reaction $\pi^- + p \to \Lambda^0 + K^0$ is studied in the region of the Σ^- and Σ^0 thresholds.

K^--d ABSORPTION AND A π-Σ RESONANCE. R. L. Schult,* Laboratory of Nuclear Studies, Cornell University, Ithaca, New York, and R. H. Capps, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York and Department of Physics, Northwestern University, Evanston, Illinois (Received January 30, 1961).

The branching ratios for the $K^- + d \to \pi + Y + N$ reactions are compared with those for the $K^- + p \to \pi + Y$ reactions. Certain of these deuteron branching ratios are shown to be independent of hyperon-nucleon final-state interaction and are inconsistent with the proton branching ratios. The most likely explanation of the discrepancy is the presence of an isospin-zero π-Σ resonance a few Mev below the $K^- + p$ threshold. This resonance is consistent with the (b-) set of K^--p scattering lengths determined by Dalitz and Tuan.

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